JC07 Rec'd PCT/PTO 2 8 MAR 2001

FORM PTO-1390 (REV 10-94)

U S DEPARTMENT OF COMMERCE PATENT AND TRADEMARK OFFICE

TRANSMITTAL LETTER TO THE UNITED STATES DESIGNATED/ELECTED OFFICE (DO/EO/US) CONCERNING A FILING UNDER 35 U.S.C. 371

ATTORNEY'S DOCKET NUMBER: 9320.125USWO

U S APPLICATION NO (If known, see 37 C F R 1 5): Unknown 09/806183

INTERNATIONAL APPLICATION NO.	INTERNATIONAL FILING DATE	PRIORITY DATE CLAIMED
PCT/FR99/02524	October 15, 1999	October 15, 1998

TITLE OF INVENTION

METHOD FOR THE ENCODING OF A SOURCE MESH WITH OPTIMIZATION OF THE POSITION OF A VERTEX RESULTING FROM AN EDGE MERGER, AND CORRESPONDING APPLICATIONS

APPLICANT(S) FOR DO/EO/US

LAURENT-CHATENET et al.

Applicant herewith submits to the United States Designated/Elected Office (DO/EO/US) the following items and other information:

1. [X] This is a **FIRST** submission of items concerning a filing under 35 U.S.C. 371.
2. [ ] This is a **SECOND** or **SUBSEQUENT** submission of items concerning a filing under 35 U.S.C. 371.
3. [X] This express request to begin national examination procedures (35 U.S.C. 371(f)) at any time rather than delay examination until the expiration of the applicable time limit set in 35 U.S.C. 371(b) and PCT Articles 22 and 39(1).
4. [X] A proper Demand for International Preliminary Examination was made by the 19th month from the earliest claimed priority date.
5. [X] A copy of the International Application as filed (35 U.S.C. 371(c)(2))
   a. [X] is transmitted herewith (required only if not transmitted by the International Bureau).
   b. [X] has been transmitted by the International Bureau.
   c. [ ] is not required, as the application was filed in the United States Receiving Office (RO/US)
6. [X] A translation of the International Application into English (35 U.S.C. 371(c)(2)).
7. [X] Amendments to the claims of the International Application under PCT Article 19 (35 U.S.C. 371(c)(3))
   a. [ ] are transmitted herewith (required only if not transmitted by the International Bureau).
   b. [ ] have been transmitted by the International Bureau.
   c. [ ] have not been made; however, the time limit for making such amendments has NOT expired.
   d. [X] have not been made and will not be made.
8. [ ] A translation of the amendments to the claims under PCT Article 19 (35 U.S.C. 371(c)(3)).
9. [X] An oath or declaration of the inventor(s) (35 U.S.C. 371 (c)(4)).
10. [X] A translation of the annexes to the International Preliminary Examination Report under PCT Article 36 (35 U.S.C. 371(c)(5)).

Items 11. to 16. below concern document(s) or information included:

11. [X] An Information Disclosure Statement under 37 CFR 1.97 and 1.98.
12. [ ] An assignment document for recording. A separate cover sheet in compliance with 37 CFR 3.28 and 3.31 is included.
13. [X] A FIRST preliminary amendment.
    [ ] A SECOND of SUBSEQUENT preliminary amendment.
14. [ ] A substitute specification.
15. [ ] A change of power of attorney and/or address letter.
16. [X] Other items or information: Translation of Amended claims; International Search Report; International Preliminary Examination Report; Form 1449, cited references

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U.S. APPLICATION NO. (If known, see 37 C.F.R. 1.5)	INTERNATIONAL APPLICATION NO	ATTORNEY'S DOCKET NUMBER
Unknown 09/806183	PCT/FR99/02524	9320.125USWO

17. [X] The following fees are submitted:

BASIC NATIONAL FEE (37 CFR 1.492(a) (1)-(5)):

Search Report has been prepared by the EPO or JPO....................**$860.00**

International preliminary examination fee paid to USPTO (37 CFR 1.492(a)(1))....................**$690.00**

No international preliminary examination fee paid to USPTO (37 CFR 1.482) but international search fee paid to USPTO (37 CFR 1.445(a)(2))....................**$710.00**

Neither international preliminary examination fee (37 CFR 1.482) nor international search fee (37 CFR 1.445(a)(3)) paid to USPTO....................**$1000.00**

International preliminary examination fee paid to USPTO (37 CFR 1.482) and all claims satisfied provisions of PCT Article 33(2)-(4)....................**$100.00**

				CALCULATIONS	PTO USE ONLY
ENTER APPROPRIATE BASIC FEE AMOUNT =				$860.00	
Surcharge of **$130.00** for furnishing the oath or declaration later than [ ] 20 [ ] 30 months from the earliest claimed priority date (37 CFR 1.492(e)).				$0	
CLAIMS	NUMBER FILED	NUMBER EXTRA	RATE		
Total claims	23 -20 =	3	X $18.00	$54.00	
Independent claims	1 -3 =	0	X $80.00	$0	
MULTIPLE DEPENDENT CLAIM(S) (if applicable)			+ $260.00	$0	
TOTAL OF ABOVE CALCULATIONS =				$914.00	
Reduction by 1/2 for filing by small entity, if applicable. Small entity status is claimed pursuant to 37 CFR 1.27				$0	
SUBTOTAL =				$914.00	
Processing fee of **$130.00** for furnishing the English translation later than [ ] 20 [ ] 30 months from the earliest claimed priority date (37 CFR 1.492(f). +				$0	
TOTAL NATIONAL FEE =				$914.00	
Fee for recording the enclosed assignment (37 CFR 1.21(h)). The assignment must be accompanied by an appropriate cover sheet (37 CFR 3.28, 3.31). $40.00 per property +				$0	
TOTAL FEES ENCLOSED =				$914.00	
				Amount to be: refunded	$0
				charged	$0

a. [X] Check in the amount of $914.00 to cover the above fees is enclosed.

b. [ ] Please charge my Deposit Account No. ______________ in the amount of $ ____________ to cover the above fees. A duplicate copy of this sheet is enclosed.

c. [X] The Commissioner is hereby authorized to charge any additional fees which may be required, or credit any overpayment to Deposit Account No. 13-2725.

NOTE: Where an appropriate time limit under 37 CFR 1.494 or 1.495 has not been met, a petition to revive (37 CFR 1.137(a) or (b)) must be filed and granted to restore the application to pending status.

SEND ALL CORRESPONDENCE TO
John J. Gresens
MERCHANT & GOULD
P.O. Box 2903
Minneapolis, MN 55402-0903

SIGNATURE: [signature]

NAME: John J. Gresens

REGISTRATION NUMBER: 33,112

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE 09/806183

Applicant: LAURENT-CHATENET et al.

Docket: 9320.125USWO

Title: METHOD FOR THE ENCODING OF A SOURCE MESH WITH OPTIMIZATION...

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CERTIFICATE UNDER 37 CFR 1.10
'Express Mail' mailing label number. EL658339235US
Date of Deposit. March 28, 2001
I hereby certify that this paper or fee is being deposited with the United States Postal Service 'Express Mail Post Office To Addressee' service under 37 CFR 1.10 and is addressed to the Assistant Commissioner for Patents, Washington, D.C 20231.

By: [signature]
Name: Brant Miles

BOX PATENT APPLICATION
Assistant Commissioner for Patents
Washington, D.C. 20231

Sir:

We are transmitting herewith the attached:

- ☒ Transmittal sheet, in duplicate, containing Certificate under 37 CFR 1.10.
- ☒ National Stage PCT Patent Application: Spec. 35 pgs; 23 claims; Abstract 1 pg. The fee has been calculated as shown below in the 'Claims as Filed' table.
- ☒ 12 sheets of formal drawings
- ☒ An unsigned Combined Declaration and Power of Attorney
- ☒ A check in the amount of $914.00 to cover the Filing Fee
- ☒ Other: Preliminary Amendment; Information Disclosure Statement; form 1449, cited references; Translation of Amended claims; International Search Report; International Preliminary Examination Report; PTO-1390
- ☒ Return postcard

CLAIMS AS FILED

Number of Claims Filed		In Excess of:		Number Extra		Rate		Fee
Basic Filing Fee								$860.00
Total Claims								
23	-	20	=	3	x	18.00	=	$54.00
Independent Claims								
1	-	3	=	0	x	80.00	=	$0.00
MULTIPLE DEPENDENT CLAIM FEE								$0.00
TOTAL FILING FEE								$914.00

Please charge any additional fees or credit overpayment to Deposit Account No. 13-2725. A duplicate of this sheet is enclosed.

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(612) 332-5300

23552
PATENT TRADEMARK OFFICE

By: [signature]
Name: John J. Gresens
Reg. No.: 33,112
Initials: JJG/tvm

(PTO TRANSMITTAL - NEW FILING)

09/806183

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S/N unknown

PATENT

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

Applicant:	LAURENT-CHATENET et al	Docket No.:	9320.125USWO
Serial No.:	unknown	Filed:	concurrent herewith
Int'l Appln No.:	PCT/FR99/02524	Int'l Filing Date:	October 15, 1999
Title:	METHOD FOR THE ENCODING OF A SOURCE MESH WITH....		

CERTIFICATE UNDER 37 CFR 1.10
'Express Mail' mailing label number: EL658339235US
Date of Deposit: March 28, 2001
I hereby certify that this correspondence is being deposited with the United States Postal Service 'Express Mail Post Office To Addressee' service under 37 CFR 1.10 on the date indicated above and is addressed to the Assistant Commissioner for Patents, Washington, D.C . 20231.
By: Brant Miles
Name: Brant Miles

PRELIMINARY AMENDMENT

Box PCT
Assistant Commissioner for Patents
Washington, D. C. 20231

Dear Sir:

In connection with the above-identified application filed herewith, please enter the following preliminary amendment, which is based on the Article 34.2 amendments, based on claims amended in prosecution of the international application and published in the International Preliminary Examination Report, a copy of which is enclosed herewith (marked-up copy attached):

IN THE SPECIFICATION

A courtesy copy of the present specification is enclosed herewith. However, the World Intellectual Property Office (WIPO) copy should be relied upon if it is already in the U.S. Patent Office.

IN THE CLAIMS

Please amend the following claims:

3. (Amended) Method for the simplification of a source mesh according to claim 1, characterized in that it comprises a step for the selection of an edge merger to be made among all the edge mergers possible, taking account of:

- at least one piece of information representing the curvature defined locally around the edge considered ;
- at least one piece of information representing the geometrical dynamics defined locally.

6. (Amended) Method for the simplification of a source mesh according to claim 1, characterized in that said information representing the geometrical dynamics belongs to the group comprising :

- the length of the edge considered ;
- a mean of the surfaces of the faces neighboring said edge considered ;
- a mean of the lengths of the edges adjacent to the vertices forming said edge considered ;
- a combination of the lengths of edges and/or surfaces of faces ;

7. (Amended) Method for the simplification of a source mesh according to claim 1, characterized in that the decimation is interrupted as a function of one of the criteria belonging to the group comprising :

- a compression rate achieved ;
- a geometrical complexity achieved, expressed by a number of vertices or faces ;
- a threshold of curvature achieved.

8. (Amended) Method for the simplification of a source mesh according to claim 1, characterized in that it constitutes a step of initialization of a method of geometrical optimization of a mesh.

9. (Amended) Method for the geometrical optimization of a source mesh comprising a step of initialization implementing the method of simplification according to claim 1.

10. (Amended) Method for the encoding of a source mesh (M) according to claim 9, representing a 3D object, delivering a simplified mesh (M') corresponding to said source mesh (M), said meshes being defined by a set of vertices, edges and/or faces, characterized in that it implements the method for the simplification of a source mesh (M) representing a plurality of surfaces defined by vertices, faces and orientations of these faces, said method implementing a step of decimation by edge merger, consisting of the association of an edge to be decimated, defined by two vertices, with a single vertex so as to obtain a simplified mesh M',

characterized in that the method comprises a pseudo-optimizing step after said step of decimation by merger of a edge, positioning the vertex resulting from said merger so as to reduce the geometrical deviation between said source mesh M and said simplified mesh M', and then a step of minimization of a volume contained between said source mesh (M) and said simplified mesh (M').

12. (Amended) Method for the encoding of a source mesh according to claim 10, characterized in that said minimizing step implements an iterative process progressively optimizing the positions of the vertices of said simplified mesh (M').

14. (Amended) Method for the encoding of a source mesh according to claim 11, characterized in that said step of minimization implements an adaptive gradient method.

16. (Amended) Method for the encoding of a source mesh according to claim 10 characterized in that, at each iteration, an elementary variation of said volume corresponding to a vector field $\overrightarrow{\delta M}$ is determined and in that, since the surface is parametrized by u and v so that a vector $\overrightarrow{\delta M}$ is expressed in the form $\overrightarrow{\delta M}$ (u, v), said elementary variation is likened to the parallelepiped generated by the evolution of the surface area element dudv in the direction $\overrightarrow{\delta M}$ (u, v).

17. (Amended) Method for the encoding of a source mesh according to claim 10, characterized in that said simplified mesh is parametrized by means of a model of finite elements.

19. (Amended) Method for the encoding of a source mesh according to claim 10, characterized in that said method implements a progressive encoding of said simplified mesh by decimation and local optimization.

20. (Amended) Method for the encoding of a source mesh according to claim 10, characterized in that it comprises a step of limitation of the deterioration due to an elementary conversion implementing a priority queue on the elementary conversions.

23. (Amended) Application of the method for the encoding of a source mesh according to claim 1 to at least to the following fields :

- virtual reality ;
- scientific simulation ;
- modelling.

REMARKS

The above preliminary amendment is made to remove multiple dependencies from claims 3, 6, 7, 8, 9, 10, 12, 14, 16, 17, 19, 20 and 23.

Applicants respectfully request that the preliminary amendment described herein be entered into the record prior to calculation of the filing fee and prior to examination and consideration of the above-identified application.

If a telephone conference would be helpful in resolving any issues concerning this communication, please contact Applicants' primary attorney-of record, John J. Gresens (Reg. No. 33,112), at (612) 371.5265.

Respectfully submitted,

MERCHANT & GOULD P.C.
P.O. Box 2903
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Dated: March 28, 2001

By [signature]
John J. Gresens
Reg. No. 33,112

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3. (Amended) Method for the simplification of a source mesh according to [any of the claims 1 and 2] claim 1, characterized in that it comprises a step for the selection of an edge merger to be made among all the edge mergers possible, taking account of :

- at least one piece of information representing the curvature defined locally around the edge considered ;
- at least one piece of information representing the geometrical dynamics defined locally.

6. (Amended) Method for the simplification of a source mesh according to [any of the claims 1 to 5] claim 1, characterized in that said information representing the geometrical dynamics belongs to the group comprising :

- the length of the edge considered ;
- a mean of the surfaces of the faces neighboring said edge considered ;
- a mean of the lengths of the edges adjacent to the vertices forming said edge considered ;
- a combination of the lengths of edges and/or surfaces of faces.

7. (Amended) Method for the simplification of a source mesh according to [any of the claims 1 to 6] claim 1, characterized in that the decimation is interrupted as a function of one of the criteria belonging to the group comprising :

- a compression rate achieved ;
- a geometrical complexity achieved, expressed by a number of vertices or faces ;

- a threshold of curvature achieved.

8. (Amended) Method for the simplification of a source mesh according to [any of the claims 1 to 7] claim 1, characterized in that it constitutes a step of initialization of a method of geometrical optimization of a mesh.

9. (Amended) Method for the geometrical optimization of a source mesh comprising a step of initialization implementing the method of simplification according to [any of the claims 1 to 7] claim 1.

10. (Amended) Method for the encoding of a source mesh (M) according to claim 9, representing a 3D object, delivering a simplified mesh (M') corresponding to said source mesh (M), said meshes being defined by a set of vertices, edges and/or faces, characterized in that it implements [a step of simplification according to any of the claims 1 to 7] the method for the simplification of a source mesh (M) representing a plurality of surfaces defined by vertices, faces and orientations of these faces, said method implementing a step of decimation by edge merger, consisting of the association of an edge to be decimated, defined by two vertices, with a single vertex so as to obtain a simplified mesh M',

characterized in that the method comprises a pseudo-optimizing step after said step of decimation by merger of a edge, positioning the vertex resulting from said merger so as to reduce the geometrical deviation between said source mesh M and said simplified

mesh M', and then a step of minimization of a volume contained between said source mesh (M) and said simplified mesh (M').

12. (Amended) Method for the encoding of a source mesh according to [any of the claims 10 and 11] claim 10, characterized in that said minimizing step implements an iterative process progressively optimizing the positions of the vertices of said simplified mesh (M').

14. (Amended) Method for the encoding of a source mesh according to [any of the claims 11 to 13] claim 11, characterized in that said step of minimization implements an adaptive gradient method.

16. (Amended) Method for the encoding of a source mesh according to [any of the claims 10 to 15] claim 10 characterized in that, at each iteration, an elementary variation of said volume corresponding to a vector field $\overrightarrow{\delta M}$ is determined and in that, since the surface is parametrized by u and v so that a vector $\overrightarrow{\delta M}$ is expressed in the form $\overrightarrow{\delta M}$ (u, v), said elementary variation is likened to the parallelepiped generated by the evolution of the surface area element dudv in the direction $\overrightarrow{\delta M}$ (u, v).

17. (Amended) Method for the encoding of a source mesh according to [any of the claims 10 to 16] claim 10, characterized in that said simplified mesh is parametrized by means of a model of finite elements.

19. (Amended) Method for the encoding of a source mesh according to [any of the claims 10 to 18] claim 10, characterized in that said method implements a progressive encoding of said simplified mesh by decimation and local optimization.

20. (Amended) Method for the encoding of a source mesh according to [any of the claims 10 to 19] claim 10, characterized in that it comprises a step of limitation of the deterioration due to an elementary conversion implementing a priority queue on the elementary conversions.

23. (Amended) Application of the method for the encoding of a source mesh according to [any of the claims 1 to 22] claim 1 to at least to the following fields :

- virtual reality ;
- scientific simulation ;
- modelling.

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METHOD FOR THE ENCODING OF A SOURCE MESH WITH OPTIMIZATION OF THE POSITION OF A VERTEX RESULTING FROM AN EDGE MERGER, AND CORRESPONDING APPLICATIONS

1. FIELD OF THE INVENTION AND APPLICATIONS

1.1 Field of the invention

The field of the invention is that of the encoding of geometrical data structures or meshes, especially large-sized meshes. More specifically, the invention relates to the representation and encoding of objects or scenes in three dimensions. More specifically again, the invention relates to a technique of approximation of a 3D source mesh that can be used alone or in combination with other known techniques. In the latter case, the method of the invention may be an advantageous step of initialization.

A mesh is conventionally defined by a set of vertices and oriented faces defining a topology. Meshes of this kind are for example used in computer graphics to model 3D objects with a limited geometrical complexity.

The approximation of a mesh M is done by finding a mesh M' that has a geometrical complexity that is lower than the geometrical complexity of the mesh M and approaches the geometry of M to the utmost possible extent.

1.2 Exemplary applications

The invention can be applied in all fields where it is desirable to reduce the number of information elements needed to represent and/or efficiently manipulate a 3D object or a set of objects, for example to analyze it, store and/or transmit it and/or provide for its rendition.

By way of an indication, the invention can be applied especially to the fields of :

- virtual reality (visits or virtual shops, leisure activities, remote handling, etc.). In this type of application, the approximation of the meshes reduces the cost of rendition of complex scenes, especially by defining the notion of scaleability on meshes (in terms of the function of the viewpoint, graphic capacities, desired refresh rate, etc.). In the case of distributed or shared virtual reality, this can also be used to adapt the complexity of a scene to the rendition and storage capacities of the different terminals, as well as to the bit rates of the networks ;

- scientific simulation (finite elements, CAO, etc.). The reduction of the geometrical complexity of the models brings accelerated computation time and faster decision-making, especially in CAO designing, and eliminates redundant information in a 3D database;
- modelling (3D scanner with reconstruction of surfaces from non-organized points, volume scanners, reconstruction of surfaces from stereoscopic photos or video sequences, digital models of terrains with satellite imaging or radar, etc.). A digital terrain model can thus be used to obtain a mesh representing the topology of a region. A mesh of this kind is obtained by the regular sampling of an image storing the altitude information at each point. The result thereof is a large quantity of information comprising information that is useless for scientific simulation or too costly for rendition (in the case of simulators). The approximation of meshes reduces the quantity of data while ensuring high geometrical fidelity to the initial data and the preservation of the topology.

2. PRIOR ART

2.1 The families of algorithms

Several techniques of mesh approximation are already known. The most widespread meshes may be classified under three great families of algorithms depending on whether they work by:

- decimation;
- sub-critical resampling;
- adaptive subdivision.

2.1.1 Decimation

Decimation consists in iteratively withdrawing vertices and/or faces from a mesh. This operation is called an elementary simplification operation. The methods that implement this principle of decimation may also optimize the positions of the vertices after or during the simplification, the latter being chosen so as to preserve the topology of the mesh to the utmost extent.

2.1.2 Sub-critical re-sampling

The re-sampling consists in sampling an original model, either by taking points randomly on its surface and then retriangulating or by defining a 3D grid and agglomerating the vertices in each elementary box of the grid.

The model thus generated is simplified and must approximate the initial data as closely as possible. This is a fast technique but preserves neither the topology nor the visually important characteristics of the meshes.

2.1.3 Adaptive subdivision

Adaptive subdivision starts with a model comprising a very simple geometry that is then recursively subdivided by adding a detail at each iteration in the regions in which the approximation error is the maximum.

2.1.4 Combinations of algorithms

In order to enable an approximation of a mesh with a satisfactory quality of reconstruction, it is necessary to combine a decimation and an optimization of the positions of the vertices preserved. In other words, since the basic goal of a source mesh encoding method is to maximize the quality of the approximation for a given geometrical complexity, this goal must have the following properties:

- decimation;
- preservation of the topology;
- optimization of the positions according to a predefined error criteria.

There is also a first known method called "remeshing" that meets these criteria. It has been presented especially in the document by Greg Turk "Retiling Polygonal Surfaces", SIGGRAPH 92 Conference proceedings, pp. 55-64, 1992). It works by sampling, decimation and optimization of positions. It can be used to parametrize the number of desired levels of resolution and the final number of vertices for each of them.

The algorithm starts with a random distribution of the number of vertices parametrized on the surface of the model and then optimizes their position by binding them by forces of repulsion (functions of local curvature). It then triangulates the polygons thus formed by including the vertices of the original mesh. Finally, the model is decimated one vertex after the other (if this operation keeps the local topology) and then the resulting aperture is retriangulated.

Thus, a refined model is obtained in the regions of high curvature and a decimated model is obtained in the plane regions.

One drawback of this method is that it has a random part. Consequently, two successive decimations do not give the same result. Furthermore, this technique tends to smoothen the discontinuities in most situations.

Furthermore, fidelity to initial data is not expressed directly during the optimization of the positions.

Another technique known as "progressive mesh encoding" has been developed by Hugues Hoppes in "Progressive Meshes", SIGGRAPH 96 Conference Proceedings, pp. 99-108, 1996. It relies on the decimation and optimization of points. This technique is described in greater detail in Appendix 1 with reference to Figures 24 to 27, so as not to burden the present discussion. It is clear however that this Appendix 1 as well as Appendix 2 form an integral part of the description.

This technique of progressive mesh encoding has various drawbacks. In particular, it does not naturally preserve the geometrical discontinuities or singularities; This means that there has to be a management of special cases such as corners, finishing sharp edges and regular sharp edges to prevent any breaks in topology. This leads to complicated computations that are difficult to implement and assume difficult parametrizing operations (what is a sharp edge? how to weight the bounce term to prevent a smoothening of the surfaces? etc.).

Furthermore, it is not efficient for characterizing the perceptual differences between two meshes.

3. GOALS OF THE INVENTION

The invention especially relates to the technique of decimation implemented for example by these different techniques of mesh encoding or by others.

The goal of the invention especially is to overcome the different drawbacks of the known techniques;

Thus, a goal of the invention is to provide a method of mesh simplification by decimation (edge merger) that is more effective in terms of perceptual quality than known techniques.

In other words, a goal of the invention is to give a method of this kind which, up to a high level of decimation, maintains the singularity of the meshes and preserves the topology.

A goal of the invention is also to give a method of this kind that is simple to implement in terms of computations to be made and has high execution speed.

According to a first aspect of the invention, another goal is to provide a method of this kind that can be used alone in order to give a fast method of mesh simplification.

According to a second aspect of the invention, one goal is to give a method of decimation of this kind that can be used, so that is improved, in a method of geometrical mesh optimization.

According to yet another aspect of the invention, a goal of the invention is to provide a method for the encoding of a source mesh in three dimensions with a better ratio between quality of approximation and geometrical complexity than known techniques;

In particular, the invention is aimed at giving a method of this kind that more faithfully keeps the major characteristics (singularities or discontinuities) that have to be preserved.

Another goal of the invention is to give a method of this kind that does not require searching for and managing special cases (such as edges and corners) or implementing specific parameters (for example criteria of recognition of a edge, thresholds, etc.) that have to be defined for each mesh.

Yet another goal of the invention is to provide a method of this kind enabling a successive reconstruction of the mesh, this mesh being very quickly recognizable in a rough representation.

It is also a goal of the invention to give a method by which it is possible to fulfil the functions laid down in the ISO-MPEG4 standardization project, namely:

- the compression of the meshes which must greatly reduce the data describing the geometry, with minimum visual loses;
- the progressive representation of a mesh, enabling the defining of a mesh in the form of a low-resolution basic mesh and a sequence of refinements;
- the matching of a wide range of bit rates, the quality being acceptable, but for very low bit rates.

4. MAIN CHARACTERISTICS OF THE INVENTION

4.1 Study of optimization of the position of the vertices

These goals as well as others that shall appear more clearly hereinafter are achieved according to the invention by means of a method for the simplification of a source mesh (M) representing a 3D object, delivering a simplified mesh (M') corresponding to said source mesh (M), said meshes being defined by a set of vertices, edges and/or faces, said method implementing a step of decimation by edge merger, wherein with a edge to be decimated, defined by two vertices, there is associated a single vertex so as to obtain a simplified mesh M'.

This method comprises an optimizing or pseudo-optimizing step after said step of decimation by merger of a edge, providing for the positioning of the vertex resulting from said merger so as to reduce the geometrical deviation between said source mesh M and said simplified mesh M'.

Thus, the particular aspects, especially the sharp edges of the mesh are complied with.

Said step of pseudo-optimization may advantageously consist in enumerating the sharp edges around two vertices forming the edge to be merged and distinguishing the following two cases:

- if the numbers of sharp edges are the same around two vertices, the vertex resulting from the merger is placed in the middle of the segment linking said vertices;
- if the numbers of sharp edges are different, the vertex resulting from the merger is placed on the vertex with the greatest number of sharp edges.

4.2 Advantageous characteristics of the invention

The invention can be implemented especially by means of an algorithm organized in two parts: the optimizing of positions and decimation.

Decimation is used to obtain a simplified geometrical mesh, from the original mesh, while at the same keeping the topology and a high degree of resemblance with this original mesh.

The optimizing of the positions of the vertices gives the best approximation of the volume-based metrics of the invention.

By combining decimation and optimization, it is possible to generate either a mesh in progressive form (by using alternately the decimation of a

vertex and a local optimization) or distinct levels of resolution (by using successively the decimation of a set of vertices and total optimization).

Advantageously, the method of the invention comprises a step for the selection of one edge merger to be made among all the edge mergers possible, taking account of :

- at least one piece of information representing the curvature defined locally around the edge considered;
- at least one piece of information representing the geometrical dynamics defined locally.

By taking these two criteria into account it is possible, as shall be seen hereinafter, to optimize the choice of the mergers to be made, in eliminating the perceptually less significant elements as a priority.

Preferably, said step of selection implements a queue of priorities of edges to be merged as a function of a priority criterion, said information representing the curvature and then a secondary criterion, said information representing the geometrical dynamics.

This hierarchy of criteria is used to achieve high efficiency.

Advantageously, said selection step manages a threshold of curvature, only the edges with a curvature smaller than said threshold being considered for the application of said secondary criterion, said threshold being increased when there is no longer any edge having a curvature below this threshold.

According to other particular embodiments, said information representing the geometrical dynamics may belong to the group comprising:

- the length of the edge considered;
- a mean of the surfaces of the faces neighboring said edge considered;
- a mean of the lengths of the edges adjacent to the vertices forming said edge considered;
- a combination of the lengths of edges and/or surfaces of faces;
- any other characteristic magnitude connected to the local density.

As shall be seen hereinafter, taking into account the length of the edge is a simplified technique and gives very good results.

The decimation may be interrupted in particular as a function of one of the criteria belonging to the group comprising :

- a compression rate achieved;
- a geometrical complexity achieved, expressed by a number of vertices or faces;
- a threshold of curvature achieved.

According to a first embodiment of the invention, the method of simplification of a source mesh forms a step of initialization of a method of geometrical optimization of a mesh.

The invention also relates to a method of geometrical optimization of this kind of a source mesh comprising a step of initialization implementing the method of simplification described here above.

According to a second mode of implementation of the invention, the method of simplification of a source mesh may be used alone.

Thus, the method of mesh encoding may implement a step of minimization of a volume contained between said source mesh (M) and said simplified mesh (M').

Thus, the invention relates to a technique of approximation of meshes using a volume-based metrics and not, as is conventionally the case, a metrics based on the distance between a vertex and a surface (a distance of this kind furthermore is not unique).

This new approach naturally takes account of the major characteristics to be kept (singularities) without it being necessary to have recourse to the detection of special cases. No particular parametrizing is required before the processing of a mesh. The volume between two meshes is simply minimized.

Advantageously, since each of said meshes is defined by the position of each of the vertices and edges that connect them, said minimizing step provides for the determining of the position of the vertices (X_1, X_2, ..., X_n) of said simplified mesh (M') minimizing the volume V(M, M') between said source mesh and said simplified mesh.

Preferably, said minimizing step implements an iterative process progressively optimizing the positions of the vertices of said simplified mesh (M').

Said iterative process may especially be interrupted when at least one of the following stopping criteria is achieved:

- a maximum number (N) of iterations;

- a difference between two successive shift vectors of the positions of the vertices that is smaller than a predetermined threshold (ε).

According to an advantageous embodiment, said step of minimization implements an adaptive gradient method. This method, as shall be seen hereinafter, implements the invention with a reduced complexity of computation. Indeed, it is no longer necessary to carry out an explicit computation of volume.

In this case, said adaptive gradient method may advantageously rely on the following operations:

- the selection of a vector X_p of $\mathbf{R}^{3n}$ ($n \geq 1$) of said simplified mesh and computation of the gradient $\nabla E(X_p)$ in X_p of the function to be minimized $E = d(M,M')$:
- the determining of the position of X_p^* of X_p of said mesh according to the relationship defined in the iteration k+1 by:

$$X_p^{k+1} = X_p^{k} - \gamma_k \times \frac{\nabla E(X_p^{k})}{\left\| \nabla E(X_p^{k}) \right\|}$$

k varying from 0 to n-1 (with $n < N$) and γ_k being the step of said relationship :

N represents the number of useful iterations to attain the convergence (this convergence being achieved when the difference between two successive determinations is smaller than a threshold (ε) or when a maximum number of iterations is achieved).

It may be noted that X_p belongs to $\mathbf{R}^3$ if a local optimization is done or to $\mathbf{R}^{3n}$ if a total optimization is done. In other words, the positions of a vertex are optimized in the event of local optimization and the positions of all the vertices are optimized simultaneously (in the form of a singe vector of $\mathbf{R}^{3n}$) in the case of a total optimization.

Preferably, said step γ_k is variable and varies as a function of the oscillation of two successive shifts of the vertices and/or as a function of the variations of the energy. It may be maintained especially between two terminals γ_{min} and γ_{max}.

According to an advantageous embodiment of the invention, at each variation, an elementary variation of said volume corresponding to a vector

field $\delta\vec{M}$ is determined. Since the surface is parametrized by u and v, so that a vector $\delta\vec{M}'$ is expressed in the form $\delta\vec{M}$ (u, v), said elementary variation is likened to the parallelepiped generated by the evolution of the surface area element dudv in the direction $\delta\vec{M}'$ (u, v).

This elementary variation of the volume between M and M' may advantageously be written as follows:

$$\delta d(M,M',\delta\vec{M}') = -\int\int_{u,v} \eta(u,v)\vec{n}(u,v)\delta\vec{M}'(u,v)d\sigma(u,v)$$

With : $\eta(u, v) = \eta(M, M', \vec{n}(u,v)) = 1$ if the unit volume $\vec{n}(u,v)$ is oriented towards the interior volume and –1 if not.

According to a preferred embodiment of the invention, said simplified mesh is parametrized by means of a model of finite elements. Said finite elements are advantageously obtained by means of a refined interpolator.

In this case, said simplified mesh can be written as follows:

$$M(u,v) = \sum_{i=1}^{N} X_i \lambda_i(u,v)$$

with $\lambda_i(u,v)$ being a shape function matched to a model of triangular finite elements;

and X_i being the vertex of said mesh defined in $\mathbf{R}^3$,

and a shape function is defined on the mesh by means of barycentric coordinates.

By using the previous expression, we get:

$$\delta d(M,M',\delta X_1,\ldots,\delta X_n) = -\int\int_{u,v} \eta(u,v)\vec{n}(u,v)\sum_{i=1}^{N}\delta X_i\lambda_i(u,v)d\sigma(u,v)$$

$$= -\sum_{i=1}^{N}\int\int_{u,v} \eta(u,v)\vec{n}(u,v)\delta X_i\lambda_i(u,v)d\sigma(u,v)$$

Advantageously, the position of a vertex X_i at the k-th iteration is written as follows:

$$X_i^{k+1} = X_i^k - \gamma_k \frac{\partial\,\delta d}{\partial X_i}$$

with the partial derivative of the distance for a vertex X_i :

$$\frac{\partial\,\delta d}{\partial X_i} = -\iint_{u,v\,\in\,\mathrm{Supp}(\lambda_i)} \eta(u,v)\vec{n}(u,v)\lambda_i(u,v)d\sigma(u,v)$$

Advantageously, the method of the invention implements a progressive encoding of said simplified mesh by decimation and local optimization. Thus, said simplified mesh is advantageously represented by a basic mesh and a sequence of refinements of said basic mesh.

Advantageously, the computation of the gradient at the k-th iteration comprises the following steps:

- discretizing of the expression of the partial derivative of the distance for each vertex X_i in the form :

$$\frac{\partial\,\delta d}{\partial X_i} = -\sum_{\tau\in S}\sum_{i_\tau=1}^{N_\tau}\sum_{j_\tau=1}^{M_\tau} \eta(i_\tau,j_\tau)\vec{n}(i_\tau,j_\tau)\lambda_i(i_\tau,j_\tau)d\sigma(i_\tau,j_\tau)$$

with:

- S as the set of triangles neighboring the vertex X_i ;
- N_τ the number of points sampled in the direction of u ;
- M_τ the number of points sampled in the direction of v ;
- computation of the orientation of the surfaces by identification of the closest intersection with the source mesh M.

According to a preferred aspect of the invention, the relative orientation of the surfaces with respect to said source mesh and said simplified mesh is computed according to the equation:

$$\eta(u,v) = -<\vec{n}_{M'},\vec{n}_M> . <\vec{n}_{M'}, \overline{X_j X_M}>$$

with :

X_j as the point sampled by M' ;

X_M the point of intersection of the straight line passing through X_j and having the direction $\vec{n}$ with the source mesh M ;

$\vec{n}_M$ the normal to the source mesh M at the point X_M ;

$\vec{n}_{M'}$ the normal to the source mesh M' at the point X_j ;

($\overline{X_j X_M}$ represents a vector).

According to another advantageous aspect of the invention, the method comprises a step of limitation of the deterioration due to an

elementary conversion implementing a priority queue on the elementary conversions.

Preferably, said step of limitation of the deterioration due to an elementary conversion comprises the steps of:

- computing a cost for each possible elementary conversion ;
- carrying out the lowest cost elementary conversion ;
- recomputing the costs of the elementary conversions modified by the previous elementary conversion ;
- adding the new elementary conversions created and computation of the corresponding costs.

The cost of an elementary conversion (T_i) may be expressed by:

$$C(T_i(X_i, X_j)) = \max d_2(V_M, F(X_i^f))$$

with :

T_i conversion merging two vertices X_i and X_j of said simplified mesh M' ;

X_i^f the vertex of said simplified mesh M' resulting from said conversion;

$F(X_i^f)$ the faces of said simplified mesh M' neighboring the vertex X_i^f after said conversion;

V_M set of the vertices of said source mesh M belonging to the faces having been intersected during the computation of the orientation of the surfaces during said minimization.

The method of the invention can be applied to a very large number of technical fields and especially at least to the fields belonging to the group comprising:

- virtual reality ;
- scientific simulation ;
- modelling.

5. LIST OF FIGURES

Other features and advantages of the invention shall appear from the following description of a preferred embodiment of the invention given by way of simple illustrative and non-restrictive examples and the appended figures, of which:

- Figure 1 illustrates the principle of a edge merger ;

- Figure 2 illustrates the principle of the priority queue combining the curvature and the geometrical dynamics defined locally according to the invention ;

- Figure 3 illustrates the enumeration of the sharp edges around a vertex of the mesh ;

- Figure 4 shows the principle of the pseudo-optimization between edge merger according to the invention ;

- Figure 5 illustrates the principle consisting in minimizing the volume between two surfaces ;

- Figures 6 to 11 are discussed in Appendix 2 describing the mathematical aspects of the invention and respectively illustrating :

- Figure 6 : elementary variation of the mesh ;

- Figure 7 : elementary variation of the mesh in the direction of the normal ;

- Figure 8 : system of coordinates on a triangle ;

- Figure 9 : shape function ;

- Figures 10 and 11 : examples of the surface developments on two curvatures in two dimensions ;

- Figure 12 illustrates the updating of the face adjacencies after merger of a edge adjacent to two faces ;

- Figure 13 illustrates the updating of the face adjacencies of the merger of a edge adjacent to one face ;

- Figure 14 illustrates the sampling on a triangle for numerical integration ;

- Figure 15 illustrates the principle of orientation pertaining to the surfaces so as to prevent the formation of folds on the surface when the initial position is too far distant from the optimum ;

- Figure 16 illustrates the cost of an elementary conversion ;

- Figure 17 is a block diagram of the approximation of meshes according to the invention ;

- Figure 18 is a block diagram of the optimization of a vertex X_i ;

- Figures 19 to 22 illustrate the behavior of the method of the invention on a rectangular parallelepiped after decimation on a corner (Figure 19), a regular sharp edge (Figures 20 and 21) and a plane (Figure 22) ;

- Figure 23 illustrates the different iterations (12) performed on a simple example according to the invention ;

- Figures 24 to 27 are discussed in Appendix 1 presenting the known technique of progressive mesh encoding, and respectively illustrate :

- Figure 24 (also commented hereinafter): elementary conversion (edge merger) ;

- Figure 25 : initial positions;

- Figure 26 : updating the conversions in the vicinity affected by the last modification ;

- Figure 27 : adding the newly created conversions.

6. GENERAL PRINCIPLE OF THE IMPLEMENTATION OF THE INVENTION

6.0 Introduction

According to the main aspect of the invention, there is provided a step for the optimization of the position of the vertices (§ 6.2) that comes within in the framework of a edge merger chosen for example according to the technique described in § 6.1. This can then be implemented within the framework of the methods of § 6.3 and 6.4.

6.1 Geometrical simplification

As indicated here above, the invention relates especially to a new technique of simplification of a 3D mesh relying on the implementation of a priority queue combining the local curvature and the local geometrical dynamics. This technique especially has the advantage of preserving the singularities on the meshes up to a high level of decimation. It furthermore has a valuable execution speed.

According to one aspect of the invention, a priority queue managing the edge merger topological operator is constructed. This priority queue combines the criteria of local curvature and of local geometrical dynamics in order make the maximum use of the degree of freedom given by the order of the conversions to be made on the mesh.

The simplification of a mesh M consists of the construction of a mesh M' with a limited geometrical complexity that preserves a low geometrical deflection with M.

The algorithm of the geometrical simplification must enable the specifying of a geometrical resolution to the nearest vertex. For this purpose, an elementary topological operation of simplification is chosen with the following good properties : conservation of topology to a certain extent of decimation and absence of creation of holes on the surfaces and preservation of the orientations.

This elementary topological operator is the edge merger as defined for example by Hoppe (already quoted) and illustrated in Figure 1.

The merger of edges 10 consists in merging two adjacent vertices 11 and 12 into one vertex 13, eliminating the two faces 14 and 15 and positioning the vertex 13 resulting from the merger. It will be noted that this conversion is reversible (with the possibility of insertion 16 of a vertex).

Each elementary conversion decimates the mesh approximating M'. The quality of the approximation therefore deteriorates during the decimation or remains at best invariant. In order to limit the deterioration of the mesh, it is sought of course to carry out first of all conversions that have the least possible effect on the model.

For this purpose, a priority queue is defined. This priority queue contains all the conversions that can be made on the mesh (namely approximately the number of edges). During the decimation, the least cost conversion of the priority queue is done and then eliminated from the queue. The cost in the neighborhood modified by the previous operation is then recomputed and the new potential conversions on the mesh are inserted into the priority queue after their cost has been computed.

The geometrical singularities which are highly informative parts, must be preserved as long as possible during a decimation. In particular, the regions of high curvature appear to be highly informative. Consequently, the first criterion of sorting on the elementary conversions is linked to the local curvature around the edge to be fused.

The term curvature $C(X_i)$ around a vertex X_i is applied to the maximum angle between the normals to two adjacent faces around the vertex X_i. Then, the curvature around a edge (referenced by two vertices X_i and X_j) is called the mean of these criteria evaluated at each vertex.

A second criterion based on the length of the edge to be merged, which reduces the geometrical density of the mesh and gives a high appearance ratio on the resultant triangles, is implemented.

It is also possible to use a habitual formula representing compactness. However, the length of the edge to be merged has the following advantages :

- giving a mesh of uniform density in regions of neighboring curvature ;
- preserving high compactness of the triangles since this criterion will tend to crate equilateral triangles ;
- having a low cost of computation.

In other words, the priority queue according to the invention is based on simultaneously taking the following two aspects into account :

- a small triangle is valuable only in the highly informative region (namely a region of high curvature) ;
- it is desirable to reduce the density of a mesh in order to reduce its complexity.

The two criteria taken into account according to the invention are combined so as to obtain the behavior illustrated in Figure 2. This figure is a scale of the curvature graduated from 0 to π in radians.

On the regions of low curvature 21, below the first threshold 22, the density is limited and the compactness obtained is reasonable since the edges of minimum length are merged.

The threshold defining a low curvature is then increased (23) when there is no longer any conversion possible on the low curvature segment 21.

Thus, on the lowest level of decimation, the curvature constraint is automatically relaxed in order to obtain the geometrical complexity fixed.

The priority queue therefore has two levels of constraint organized in a hierarchy : the curvature thereof is the priority criterion and the density thereof is a secondary criterion.

6.2 Pseudo-optimization

After an operation of edge merger, it is possible, depending on the essential aspect of the invention, to place the vertex resulting from the merger in the optimum probable position.

For this purpose, the notion of the sharp edge is introduced. A edge is sharp when the angle formed by the perpendicular to the two adjacent faces is greater than a fixed parametrizable threshold. The number of sharp edges around the vertices of the edge to be merged is then enumerated as shown in Figure 3. The vertex 31 is not associated with any sharp edge. The vertex 32 has two sharp edges and the vertex 33 has three sharp edges.

From this enumeration, two cases are deduced for initialization as shown in Figure 4 :

- if the number of sharp edges around X_a and around X_b are identical, the initialization is done in the middle of the segment forming the edge to be merged. On the plane zones of low curvature, this makes it possible to preserve high compactness on the neighboring triangle. On a regular sharp edge (number of sharp edges equal to 2), this makes it possible to position the vertex close to the sharp edge which will be preserved during the optimization ;
- if the number of sharp edges around X_a and around X_b are different, the initialization is done on the vertex having the greatest number of sharp edges. In the most common cases, the optimum is achieved from this initial position.

In the example of Figure 4, wherein the source mesh corresponds to a parallelepiped 41, it is observed that this heuristic approach places the edge :

- on the corner of the parallelepiped when the edge to be merged forms a corner 43 ;
- on the regular sharp edge of the parallelepiped when the edge starts in the plane region and ends on the sharp edge 45.

The situations where the number of sharp edges is the same around two vertices is illustrated at 42 and 44.

6.3 Applications

As shown here above, the technique of simplification of the invention can be implemented alone to offer a mesh approximation technique or as a step of initialization of a more complete procedure of geometrical optimization such as for example the one described here below.

6.3.1 Structure

The invention can be implemented in a method of mesh approximation implementing a volume-based metric. One of the advantages of this metric is that it naturally takes account of the major characteristics to be preserved (the singularities) without its being necessary to have recourse to the detection of special cases.

In general, the approximation of meshes generates a geometrically polygonal model that is simpler than the original model. The number of polygons necessary for the representation is thus reduced while at the same time preserving the best approximation, in the perceptual sense, of the original model.

The corresponding problem may be formulated by a variational approach used to approximate a mesh M by a mesh M' (or $\hat{M}$) comprising a reduced number of triangles. This permits the shifting of the vertices by optimizing the position according to one or more criteria defined by means of an energy functional criterion to be minimized.

The approximation of a mesh generates a model that is geometrically simpler than the original model. The number of polygons needed for the representation is thus reduced while preserving the best approximation (in the perceptual sense) of the original model. The quality of the approximation defines the level of perceptual resemblance for a fixed geometrical complexity. The invention uses a volume-based metric between the "approximating mesh" and the "approximated mesh".

The principle of the algorithm is subdivided into two parts: decimation and the optimization of the positions.

The elementary operation of decimation chosen is the edge merger, described here above. The optimization algorithm chosen is an adaptive step gradient iterative algorithm in order to iteratively minimize the volume included between the model and the original mesh.

By combining decimation and optimization it is possible to generate either a mesh progressively or distinct levels of resolution.

6.3.2 Decimation

In order to simplify the geometry of a mesh, it is necessary to iteratively decimate the mesh while at the same time keeping an appropriate topology. An elementary conversion is therefore chosen : the edge merger which merges two

adjacent vertices into one vertex, eliminates two faces and places the new vertex resulting from the merger according to the above-described principle.

This decimation may be done alternately or successively with an optimization (such as the one described here above) of the positions of one or more vertices depending on the application in view. It is possible to build several levels of resolution of a mesh by using this elementary conversion, and the minimum grain sizes obtained by a decimation since the mesh is then defined to the nearest vertex. The method then uses an alternation of local decimation and local optimization of the vertex resulting from the merger.

Conversely, it is possible to define levels of resolutions in a general way by first of all decimating a set of vertices and then comprehensively optimizing the positions of the vertices of the mesh.

The successive decimation is used to construct progressive meshes. In this case, it is appropriate to make a careful choice of the order of performance of the conversion. This order is defined by means of a priority queue on the elementary conversion so as to carry out first of all the conversions that have the least possible effect on the original model.

This implies defining the notion of cost for each elementary operation and the priority queue will be sorted out by this. Thus initially, all the possible conversions are listed, the cost of each of them is computed (without making any modifications on the mesh), the low-cost conversion is made and the costs of the elementary conversions modified by the previous conversion are recomputed.

Naturally, techniques other than decimation may be used to obtain the approximate or approximated mesh, such as techniques of sub-sampling, adaptive subdivision, etc.

6.3.3 Optimization based on volume

The problem of optimization therefore consists in minimizing an error functional defined by:

$$E(M, M') = d(M, M')$$

where d(M, M') characterizes a distance between the meshes M and M' and ensures fidelity to the initial data. The optimization algorithm is chosen especially for its performance characteristics in terms of speed and/or relevance of the final solution.

An explanation is given here below of the theoretical context of the volume-based metrics used to characterize the approximation error between two meshes as

well as the variational formulation used to optimize the quality of approximation according to the invention.

6.3.3.1 Metrics

An exemplary measurement of the error or distance between the meshes M and M' is illustrated in Figure 5. The measurement of the error is given by the volume V, contained between the two surfaces formed by the meshes M and M'.

This volume is defined by the following integral of Lebesgue :

$$V(M,M') = \int_{\mathbb{R}^3} I_V(M',M)(\vec{q})d^3\vec{q}$$

with $I_v(\vec{q})$ being the function indicating the interior of the volume and $\vec{q}$ being a generic point of $\mathbf{R}^3$.

6.3.3.2 Functional

Since a mesh is partially defined by the positions of the vertices, it is possible to formulate the problem of the minimizing of the volume by means on an energy functional to be minimized, this functional being defined by:

$$E = E_{error} = d(M, M') = d(M,M',X_1,...,X_n))$$

The algorithm of optimization implemented then consists in determining the optimum positions of the vertices $X_1, X_2, ..., X_n$ that minimize the distance d(M', M).

6.3.3.3 Elementary variation in volume

The elementary progress of the mesh M' is defined by the field of vectors $\delta\vec{M'}$. An expression of the elementary variation of volume is described in Appendix 2 (§ 2).

6.3.3.4 Implementation of the adaptive gradient

The technique of optimization used to resolve the problem of minimization may advantageously rely on an iterative approach such as the adaptive gradient. Appendix 2 (§ 1) gives a detailed description of the implementation of this technique of the adaptive gradient as well as the computation of an elementary variation of volume used to determine the gradient at each iteration, and the parametrizing of the mesh by a model of triangular finite elements.

A model of triangular finite elements used to compute the gradient at each vertex is described in Appendix 2 (§ 3).

The algorithm of the gradient for a vertex X_i is written in the following form :

$$X_i^{k+1} = X_i^k - \gamma_k \frac{\partial \delta d}{\partial X_i}$$

The expression of the partial drift of the difference for a vertex X_i is:

$$\frac{\partial \delta d}{\partial X_i} = -\iint_{u,v \in Supp(\lambda_i)} \eta(u,v)\vec{n}(u,v)\lambda_i(u,v)d\sigma(u,v)$$

The support of the shape function is defined by the triangles neighboring the vertex X_i. The integral is computed by means of a discretization on the triangles (described hereinafter) and a heuristic method has been adopted to determine the relative orientation of the surfaces formed by M and M' (also described hereinafter).

At each iteration of the gradient, a shift is applied to the vertex X_i that tends to minimize the volume between the surfaces. The shift is the sum of the contributions related to the computations of orientation at several points of each triangle.

The step γ_i is adaptive. At the outset, it is initialized at γ_0 (a fraction of the length of the merged edge) and then multiplied by k (with k < 1) in the presence of an oscillation (namely when two vectors of successive shift have opposite directions:

$$<\frac{\partial \delta d^{k+1}}{\partial X_i}, \frac{\partial \delta d^{k}}{\partial X_i}> \ < 0 \quad).$$

This step γ_i is bounded (with a lower boundary of γ_{min}) in order to keep the convergence of the algorithm. We have $\gamma_i\ ]\delta, 2\lambda_0-\delta]$, with $0 < \delta < \lambda_0$ and $||HE(\lambda)|| \leq 1/\lambda_0$ (where HE represents the Hessian value of E).

We shall consider the convergence to be achieved when the shift of the vertex X_i from one iteration to the other is smaller than ε (ε being a fraction of the local dynamics of the edges of the mesh). The number of iterations of the algorithm may also be limited.

6.3.4 Description of a detailed embodiment

Since the method of the invention can be used to generate approximations of meshes at different levels of resolution, a gradual encoding of meshes by decimation and local optimization may be implemented.

To this end, an invention defines an elementary conversion of decimation adapted to progressive encoding, a method of local optimization and a priority queue on the conversions in order to maximize the ratio between quantity of information and visual quality at each iteration of the algorithm. Hereinafter, a description is

given of the implementation of this algorithm, especially the computation of the gradient defined on a vertex of the mesh.

6.3.4.1 Elementary conversion

Progressive encoding is used to define the notion of sampleability on a mesh. The maximum sampleability is obtained when the resolution of the mesh can be chosen to the nearest vertex. A mesh can thus be encoded in the form of a basic mesh (with a visually acceptable resolution) and a sequence of refinements.

In order to obtain a data structure of this kind, it is necessary to interatively decimate the mesh, locally optimize the positions of the vertices so as to locally approximate the initial mesh to the best possible extent and simultaneously record the decimation sequence in order to encode the mesh according to the refinements.

A reversible elementary conversion is therefore chosen, for example the merger of edges defined by Hoppe (document already quoted) illustrated in Figure 1. The edge merger 10 consists in merging the two adjacent vertices 11 and 12 into one vertex 13, eliminating the two faces 14 and 15 and optimizing the position of the vertex 13 resulting from the merger.

This conversion is reversible (insertion 16 of a vertex).

After conversion, the adjacencies must be updated. For this purpose, we consider a face/vertex type of structure and an adjacency defining all the relationships between the elements. The faces store the links towards their neighboring faces and each vertex lists the faces that contain it and the neighboring vertex. Figures 12 and 13 illustrate the updating of the adjacencies of the faces after merger of an edge adjacent to two faces (Figure 12) or one face (Figure 13).

The decimation is thus naturally taken into account on all the zones of the mesh without its being necessary to detect the particular cases, such as the edges adjacent to a single face.

6.3.4.2 Initialization

The optimization acts on the position of the vertex resulting from the merger of the edge formed by the vertices X_a and X_b. In order to start the optimizing algorithm with a good initial condition, it is chosen to position the vertex as closely as possible to the probable optimum. For this purpose, the notion of a sharp edge is introduced. A edge is sharp when the angle formed by the normals to the two adjacent faces is greater than a fixed parametrizable threshold. The number of sharp edges around the vertices of the edge to be merged is then enumerated according to the approach already discussed.

6.3.4.3 Optimization

It may be recalled that the expression of the functional considered:

$$E = E_{error} = d(M, M')$$

This functional is minimized by optimizing the position of the vertex X_i by a gradient algorithm:

$$X_i^{k+1} = X_i^k - \gamma_i \frac{\partial \delta d^k}{\partial X_i}$$

The determining of the step γ_i and the stopping criteria are discussed in § 6.3.3.4.

6.3.4.4 Discretization

The computation of the gradient therefore requires a discretization of the integral :

$$\frac{\partial \delta d}{\partial X_i} = -\int\int_{u,v \in \mathrm{Supp}(\lambda_i)} \eta(u,v)\vec{n}(u,v)\lambda_i(u,v)d\sigma(u,v)$$

On the support of the shape function defined by X_i.

We are thus led to sample the surface of the triangles before assessing the value of the shape function $\gamma_i(x, y, z)$ and the orientation of the surfaces $\eta(u,v)$ at each point X(,x,y,z).

The gradient is therefore expressed in the following discrete form:

$$\frac{\partial \delta d}{\partial X_i} = -\sum_{\tau \in S}\sum_{i_\tau=1}^{N_\tau}\sum_{j_\tau=1}^{M_\tau} \eta(i_\tau,j_\tau)\vec{n}(i_\tau,j_\tau)\lambda_i(i_\tau,j_\tau)d\sigma(i_\tau,j_\tau)$$

with :

- S as the set of neighboring triangles of the vertex X_i ;
- N_τ the number of points sampled in the direction u ;
- M_τ the number of points sampled in the direction v.

For a given triangle, the sampling is done in the plane passing by its edges, the number of points being proportional to the surface of the triangle, as shown in Figure 14.

The sampling reference system is formed by the edge 161 with the greatest length and the height 162 of the current triangle. Let n be the minimum number of

points per triangle fixed in advance, S_{min} the minimum area of the triangles neighboring X_i and h the sampling step. h is deduced by the following formula :

$$n \times h^2 = S_{min}$$

The shape function is assessed by a ratio of surfaces as described in Appendix 2.

6.3.4.5 Orientation of the surfaces

The relative orientation of two surfaces defined in three dimensions is difficult to define with exactness. Advantageously, a heuristic system is used to compute the orientation of the two surfaces M and M' (namely the term $\eta(u,v)$ of the equation of the gradient presented here above :

$$\eta(u,v) = -<\vec{n}_{M'},\vec{n}_M> . <\vec{n}_{M'}, \overline{X_j X_M}>$$

with:

X_j being the point sampled on M' ;

X_M the point of intersection of the straight line passing through X_j and having a direction $_{M'}$ with the source mesh M ;

$\vec{n}_M$ being the normal of the source mesh M at the point X_M ;

$\vec{n}_{M'}$ being the normal of the source mesh M' at the point X_j ;

where:

<,> represent the scalar product.

Figure 15 illustrates the computation of the orientation in two dimensions. In three dimensions, it is enough to replace the segments by triangles. This heuristic method greatly limits the formation of folds on the surface, since the term $<\vec{n}_{M'},\vec{n}_M>$ expresses the opposition of the normal when they are in opposite directions.

6.3.4.6 Priorities on the conversions

Each elementary conversion decimates the approximating mesh M'. The quality of the approximation therefore deteriorates during the decimation or at best remains invariant when the volume remains unchanged, despite the optimization of the points.

For example, if we consider a sphere, it is known that it theoretically requires an infinity of triangles to be perfectly modelled. Once the number of triangles is restricted, the quality of its approximation made to deteriorate. By contrast, in the

plane regions of an object, it is possible to decimate until an optimum mesh is obtained (thus on a parallelepiped the optimum is achieved when each face is described by two triangles).

It is progressively sought to encode the meshes and therefore iteratively remove the vertices by a edge merger operation. After the optimization of the positions, another process is therefore developed in which a priority is defined on the elementary conversions so as to first of all make the conversions that have the least possible effect on the model.

For this purpose, the notion of cost is defined for each elementary operation, and a priority queue is sorted out according to this cost.

Thus, initially all the possible conversions are listed and then the cost of each of them is computed (without making the corresponding modifications on the mesh). The lower cost conversions are made and then the costs of elementary conversions, modified by the previous conversions, are recomputed. It may be recalled that the edge merger eliminates two faces. This means that, as a consequence, the conversions pertaining to these faces are withdrawn from the priority queue. This creates one or two new conversions for which a computation is also made of the energy cost, deduced from the technique described in Appendix 2. This energy cost represents the variation in volume between the mesh before conversion and the mesh after conversion.

Since the direct computation of volume between two triangulated surfaces is complex to compute, the invention makes use of a heuristic method that can be used to simplify this problem.

Let:

- T_i be the vertex merging conversion X_i and X_j of M' ;
- X_i^f the vertex of M' resulting from the merger (for which the position has been optimized) ;
- $F(X_i^f)$ the faces of M' neighboring the vertex X_i^f after conversion ;
- V_M the set of vertices of M belonging to the faces that have been intersected during the computation of the orientation of the faces during the optimization.

The cost of an elementary conversion T_i is then expressed in the form:

$$C(T_i(X_i,X_j)) = \max d_2(V_M,F(X_i^f))$$

This cost therefore corresponds to the maximum distance from the vertices of the original mesh (beneath the faces neighboring the vertex X_i) to the neighboring

faces X_i after conversion. Figure 16 gives a 2D view of the expression of the costs in the case of a decimation on a discontinuity. As indicated here above, in three dimensions, the segments are replaced by triangles. The conversion 181 consists in merging the vertices X_i and X_j into one vertex X_i'. The vertices 182_1 to 182_4 correspond to the set V_M.

It may be noted that, in a meshed plane, the cost of a conversion would be zero, since the zones comprising a redundant information would be decimated as a priority.

Exemplary block diagrams are now presented for the implementation of the different aspects described here above.

6.3.4.7 Block diagrams

- **Approximation of meshes**

Figure 18 summarizes the approximation algorithm of a mesh M comprising n faces by a mesh M' comprising m faces (with of course m<n).

We shall start first of all by copying the source mesh M, or original mesh in the variable M', representing the simplified mesh to be determined (191). In this mesh M', a search is then made for all the elementary conversions that can be performed (192) followed by a computation (193) of the energy costs of these conversions after optimization.

The lowest cost conversion among all the possible conversions is then selected and carried out (194). This is the decimation step.

Then, an updating (195) is done of the energy costs of the conversions in the neighborhood affected by the conversion made in the step 194. Finally, the new conversions induced by the conversion made (194) are added (196) and the corresponding costs are computed.

The three steps 194, 195 and 196 are reiterated (197) n - m times until at least one of the stopping criteria is reached.

- **Positional optimization**

Figure 20 illustrates the optimizing of the vertex X_i after the merger of the edge (X_i, X_j).

A preliminary decimation is made of the following initializing parameters, partially as a function of the dynamics of the local geometry around the vertex X_i:

- Epsilon is a convergence stopping criterion that is a fraction of the mean length of the edges around the vertex X_i (typically 0.001) ;

- MaxIter is the upper terminal of the number of iterations, arbitrarily fixed (typically 200 iterations);
- StepInit, StepDec and StepMin are respectively the initializing value, the rate of decrease and the lower boundary of the step. StepInit and StepMin are fractions of the length of the edge to be merged (typically 0.1 and 0.01), and StepDec is fixed arbitrarily (typically 0.95) ;
- StepSampling is the sampling step on the triangles used for the discrete computation of the integral forming the gradient at each iteration. It is deduced from the areas of the faces neighboring the vertex X ;
- NbSharpEdges(X) is the number of sharp edges around the vertex X.

The algorithm starts with a step 201 of positional initialization, which consists of the following operations :

If (NbSharpEdge(X_i) = NbSharpEdge(X_j))
then
    initialize in the middle of the edge to be merged
else
    initialize on the greatest number of sharp edges.

A computation 202 of the gradient is then done with the discretization operations on the triangles and of orientation of the surfaces. Then, the shift 203 is done. The vertex shifts by the value of the gradient vector multiplied by the current step.

The changes 204 undergone by the step are then checked. If an oscillation is detected with the previous gradient vector, the step is multiplied by StepDec. If the step is smaller than StepMin, it is reinitialized at StepMin.

Finally, a conversion step 205 is carried out. If the distance between the current position and the previous position is smaller than Epsilon, or if the number of iterations MaxIter is reached, the algorithm is stopped (206). Else, the steps 202 to 205 are repeated (207).

6.4.3.8 Results

We shall now show, from a simple example (a parallelepiped), the behavior of the method of approximation of meshes according to the invention. The usefulness of the parallelepiped lies in the fact that it can be used to verify the quality of the approximation on the discontinuities (regular sharp edges and corners) and on the planes.

On an object of this kind, the optimum is reached when each mesh face is obtained by two triangles and the overall shape of the object is perfectly preserved.

Figure 19 illustrates the case of a decimation on a corner 211 of the parallelepiped. The initial position was placed on the corner 211 because the number of sharp edges is the maximum at this vertex (three around the corner 211 and two around the regular sharp edge of the object (vertex 212)). The optimization algorithm oscillates and then converges around this initial position 213 since the gradient step was initialized at a non-zero value and then reduced as soon as the oscillation was detected.

Figure 20 shows a decimation on a regular sharp edge. The initial position was placed in the middle of the edge 223 since the number of sharp edges is identical (two around each vertex 221 and 222). Several positions located on the regular sharp edge of the object will comply with volume invariance and the algorithm converges after oscillation around the edge on a position that depends on the initial position. The middle point 224 of the edge therefore constitutes an efficient initialization in this case.

Figure 21 illustrates another situation of decimation on a regular sharp edge in which the initial position has been placed on the sharp edge 234 since the number of sharp edges are different (two around the vertex 233 located on the edge and none around the vertex 232 located on the plane face 235 of the object). Several positions located on the regular sharp edge of the object will comply with the volume invariance and the algorithm converges after oscillation around the edge 234 on a position 236 depending on the initial position.

Figure 22 shows the case of a decimation on a plane 243. The initial position has been placed in the middle of the edge 245 formed by the vertices 241 and 242 since the number of sharp edges is identical (none around each vertex 241 and 242). Several vertices located in the plane of the object will comply with the volume invariance. The algorithm, after oscillation on either side of the plane, converges on a position depending on the initial solution. The middle point 244 of the edge therefore again constitutes an efficient initialization in this situation.

The method decimates the initial mesh by using a priority queue.

Figure 23 shows twelve successive iterations of the algorithm, numbered (b) to (m), from the original mesh numbered (a).

It is noted that the edges leading to a volume invariance are decimated as a priority. The optimum is reached for the vertices requested on this object since each face is then meshed by only two triangles.

It is observed that the edges meeting two corners of the object are not merged since this operation would lead to heavy deterioration in the quality of the approximation, since the costs and the corresponding elementary conversions are greater than the costs of a merger of edges in the plane or along a regular sharp edge.

APPENDIX 1

PROGRESSIVE MESH ENCODING

Hugues Hoppes has developed a method working by decimation and optimization of points. The decimation is achieved by a reversible elementary conversion that is easily encodable: the edge merger which assembles two adjacent vertices into one vertex, eliminates two faces and then optimizes the position of the vertex resulting from the merger (Figure 24).

The author has used a variational approach by which the process is minimized, the associated energy functional is defined by an error term and a regularization term:

$$E = E_{error} + E_{regul}$$

This error term characterizes the distance between two meshes M and $\hat{M}$ defined locally, and ensures fidelity to the initial data. The regularization term is used to comply with the topology of a mesh and provides for the unified nature of the solution. In order to initialize the problem, once the face to be eliminated has been determined, three minimizing operations (starting from three different initial positions chosen so as to accurately initialize the optimization process) are achieved (Figure 25) in order to choose the solution corresponding to the minimum energy of the functional.

Each conversion T_i that eliminates a vertex of the triangulation and modifies the mesh M_i in M_{i+1} generates an increase in the total energy that may be computed. The energy cost resulting from this decimation is as follows :

$$\begin{cases} \Delta E_{T_i} = E_{M_{i+1}} - E_{M_i} \\ \Delta E_{T_i} \geq 0 \end{cases} \qquad (1)$$

At the outset, all the possible conversions on the mesh are listed, and then the energy cost for each of them is computed. These conversions are stored in a priority queue sorted out according to their estimated cost. During the algorithm, the minimum cost conversion is done and then the cost of the conversions in the vicinity affected by this modification are updated (Figure 26). The newly created conversions are added and their estimated cost is computed (Figure 27). The algorithm therefore acts iteratively and decimates the mesh $\hat{M}$ of a vertex and of two faces at each iteration. This approach of decimation and optimization is well suited to the context of the progressive encoding since it is enough to record the sequence of the

decimations in the reverse direction (that of the refinements) to encode a mesh in a progressive form. Thus a mesh M may be represented by a mesh with the resolution M_0 and by a sequence of refinements $\{r_0, r_1, r_2, ..., r_n\}$.

The error used to characterize the distance between the two meshes is the Euclidean distance (or quadratic) distance between the point X_i to be optimized of the mesh $\hat{M}$ and the mesh M, which has to be defined. The point located at a distance D from X_i in R^3 form a sphere with a radius D :

$$d_2(X_i, M) = \min_{Y \in M} d_2(X_i, Y) = \min_{Y \in M} \|X_i - Y\|_2 \tag{2}$$

where

$$\|X\|_2 = (\sum_{j=1}^{N} X_j^2)^{1/2}, X \in \mathbb{R}^N \tag{3}$$

Since the mesh M is a triangulated surface, this generates a measurement of the distance between a point X_i and a set of triangles. It is observed that according to the definition of the L^2 in R^3 that the point of M located at the minimum distance from X_i is not always unique. The effective choice of this point depends on the order of insertion of the points and the nature of the test during the search for the minimum ($<$ or $\leq$). The metric L^2 applied to the meshes becomes the metric D^2 since the reasoning is based on a point-surface distance.

APPENDIX 2

OPTIMIZATION

The optimization technique chosen to resolve our minimization problem is an iterative method known as the adaptive gradient.

1. Adaptive gradient

Our goal therefore is to minimize the above-described functional E. Since, in every case, the stationarity of E is a necessary condition of optimality, effectively, practically all the methods of constraint-free optimization in R^N consist in searching for a stationary vector X ($\nabla E(X) = 0$). This problem is equivalent to the resolution of the system of non-linear equations:

$$\frac{\partial E}{\partial X_i}(X) = 0, \forall i = 1, ..., N \qquad (1)$$

It is possible to seek to directly resolve this system. This leads to Newton's method. However, this method may be non-convergent if the starting point of the iterations is far too distant from X. Furthermore, it assumes that the function is twice continually differentiable and requires the computation of the second derivatives at each point.

That is why the most commonly used methods proceed differently : these are iterative methods where a sequence of vectors X^0, X^1, ... are generated, converging towards a local optimum of E.

These gradient methods constitute a class of methods that proceed as follows: the procedure starts from a point X^0 and the gradient $\nabla E(X^0)$ in X^0 of the function E to be minimized is computed. Since $\nabla E(X^0)$ indicates the direction of the greatest increase in E, a shift is made by a quantity γ_0 in the direction opposite to the gradient, and the following point is defined:

$$X^1 = X^0 - \gamma_0 \times \frac{\nabla E(X^0)}{\| \nabla E(X^0 \|} \qquad (2)$$

The procedure is repeated and generates the points X^0, X^1, ..., X^n, ... according to the relationship :

$$X^{k+1} = X^k - \gamma_k \times \frac{\nabla E(X^k)}{\| \nabla E(X^k \|} \qquad (3)$$

In this class, it is necessary to report the determined-step gradient methods in which an a priori choice is made of the value of the shift γ_k. The drawback of this procedure is that the convergence may be very slow. We have therefore chosen to use the adaptive-step gradient method which consists in diminishing (and respectively increasing) the step γ_k when the error increases (and respectively diminishes) while keeping the step control constraint: $\gamma_k \in [\gamma_{min}, \gamma_{max}]$. Finally, as a stopping criterion, we have taken it as fixed that the difference between two successive errors must be below a given value ϵ .

During the minimizing, the positions of the vertices of the mesh will develop so as to minimize the volume between the two surfaces. This development requires a computation of the gradient at the iteration k (term $\nabla E(X^k)$) of the equation.

2. Elementary variation of volume

The process of optimization is therefore seen as a development of the mesh $\hat{M}$ so as to minimize $V(M,\hat{M})$. The elementary development of the mesh is defined by the field of vectors $\delta\vec{\hat{M}}$. By carrying out a parametrization of the surface by u and v, the vector $\delta\vec{\hat{M}}$ can be expressed in the form $\delta\vec{\hat{M}}(u,v)$. If the surface is locally approximated by its tangential plane, the elementary variation of volume is then the parallelepiped generated by the development of the surface area element dudv in the direction $\delta\vec{\hat{M}}(u,v)$ (Figure 6).

It is then possible to express the elementary variation of volume induced by the area element dudv : $$\delta V(M,\hat{M},\delta\vec{\hat{M}}(u,v)) = \left|\left[\frac{\partial\vec{\hat{M}}(u,v)}{\partial u} \wedge \frac{\partial\vec{\hat{M}}(u,v)}{\partial v}\right] \cdot \delta\vec{\hat{M}}(u,v)\right| \cdot dudv$$

The variation is negative when the vector $\delta\vec{\hat{M}}$ is oriented towards the interior of the error volume. Figure 7 shows the particular case where the development of the mesh is done in the direction of the normal. Let us now consider the fact that :

$$\left[\frac{\partial\vec{\hat{M}}(u,v)}{\partial u} \wedge \frac{\partial\vec{\hat{M}}(u,v)}{\partial v}\right] \cdot dudv = \vec{n}(u.v)d\sigma(u,v)$$

with $\vec{n}(u,v)$ being the unit normal at $\hat{M}(u,v)$ and $d\sigma(u,v) = \|\frac{\partial\vec{\hat{M}}}{\partial u} \wedge \frac{\partial\vec{\hat{M}}}{\partial v}\| \cdot dudv$ the surface area element.

The variation in distance between M and $\hat{M}$ is expressed as follows :

$$\delta d(M,\hat{M},\delta\vec{\hat{M}}) = \int\int_{u,v} \left|\vec{n}(u,v)\delta\vec{\hat{M}}(u,v)\right| d\sigma(u,v)$$

This expression cannot easily be used because of the absolute value. We shall now introduce the function : $\eta(u,v) = \eta(M, \hat{M}, \vec{n}(u,v))$ = 1 if the normal is oriented towards the interior volume and if not -1. We then obtain :

$$\left|\vec{n}(u,v)\delta\vec{\hat{M}}(u,v)\right| = \eta(u,v)\vec{n}(u,v)\delta\vec{\hat{M}}(u,v)$$

whence
$$\delta d(M,\hat{M},\delta\vec{\hat{M}}) = -\int\int_{u,v} \eta(u,v)\vec{n}(u,v)\delta\vec{\hat{M}}(u,v)d\sigma(u,v)$$

3. Model of triangular finite elements

In order to compute the gradient at each vertex of the mesh at the iteration k of the optimizing algorithm, the meshing is parametrized by a model of triangular finite elements P^1:

$$M(u,v) = \sum_{i=1}^{N} X_i\lambda_i(u,v) \qquad (4)$$

X_i is a vertex of the mesh defined in R^3 and $\lambda_i(u,v)$ is a form function matched with a model of triangular finite elements. A form function is defined on the mesh by means of barycentric coordinates as described in the document "The method of finite elements, basic formulation and linear problems", by O.C. Zienkiewicz and R.L. Taylor (AFNOR Technique, 4th edition, 1991). For this purpose, the vertices of the triangles (Figure 8) are numbered and a system of coordinates λ_1, λ_2 and λ_3 linked by the following relationships is defined :

$$x = \lambda_1 \cdot x_1 + \lambda_2 \cdot x_2 + \lambda_3 \cdot x_3$$

$$y = \lambda_1 \cdot y_1 + \lambda_2 \cdot y_2 + \lambda_3 \cdot y_3$$

$$1 = \lambda_1 + \lambda_2 + \lambda_3$$

We shall now determine the values of the basic functions: at the vertex 1, $\lambda_1 = 1$, $\lambda_2 = 0$ and $\lambda_3 = 0$. The λ_1 level lines are equidistant lines parallel to the side 2 - 3 along which $\lambda_1 = 0$. It is possible to express the coordinate λ_1 at a point P by a ratio of areas:

$$\lambda_1 = \frac{aire(P23)}{aire(123)} \qquad (5)$$

We obtain :

$$\lambda_1 = \frac{a_1 + b_1 \cdot x + c_1 \cdot y}{2A}$$

$$\lambda_2 = \frac{a_2 + b_2 \cdot x + c_2 \cdot y}{2A}$$

$$\lambda_3 = \frac{a_3 + b_3 \cdot x + c_3 \cdot y}{2A}$$

$$A = aire(123) = \frac{1}{2} \cdot det \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$$

With :

$$a_1 = x_2 \cdot y_3 - x_3 \cdot y_2$$

$$b_1 = y_2 - y_3$$

$$c_1 = x_3 - x_2$$

The values a_2, b_2, c_2, a_3, b_3 and c_3 are computed by circular permutation of the indices 1, 2 and 3.

4. Surface development

The development of the mesh can thus be written as follows:

$$\delta d(M, \hat{M}, \delta\vec{X}_1, \delta\vec{X}_2, \ldots, \delta\vec{X}_n) = -\sum_{i=1}^{N} \int\int_{u,v} \eta(u,v)\vec{n}(u,v)\delta\vec{X}_i\lambda_i(u,v)d\sigma(u,v)$$

the partial derivative of the distance for a vertex X_i is deduced therefrom :

$$\frac{\partial V}{\partial X_i} = -\int\int_{u,v\in Supp(\lambda_i)} \eta(u,v)\vec{n}(u,v)\lambda_i(u,v)d\sigma(u,v)$$

The gradient algorithm for a vertex X_i is written in the form:

$$X_i^{k+1} = X_i^k - \gamma_i \frac{\partial V^k}{\partial X_i}$$

where γ_i represents an adaptive step with $\gamma_i \in [\gamma_{min}, \gamma_0]$. At the outset, the step is initialized at γ_0, then multiplied by k (with k < 1) in the presence of an oscillation, namely when $\langle \frac{\partial V^{k+1}}{\partial X_i}, \frac{\partial V^k}{\partial X_i} \rangle < 0$. The behavior of the optimization algorithm may be illustrated at 2d (Figures 10 and 11), the development of the vertex X_i of the model $\hat{M}$ is linked to the sum of the vectors of orientation of the model with respect to the original curve M, weighted by the form function which is equal to 1 at X_i and decreases by 0 along the neighboring segments of X_i. At 3d, the principle is the same with a form function defined by the triangles neighboring the vertex X_i.

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AMENDED CLAIMS :

1. Method for the simplification of a source mesh M formed by a plurality of surfaces defined by vertices, faces and orientations of these faces, said method implementing a step of decimation by edge merger, consisting of the association of an edge to be decimated, defined by two vertices (11, 12), with a single vertex (13) so as to obtain a simplified mesh M',

characterized in that the method comprises a pseudo-optimizing step after said step of decimation by merger of a edge, positioning the vertex resulting from said merger as a function of a criterion taking account of a number of sharp edges around each of these two vertices forming the edge to be merged, so as to reduce the geometrical deviation between said source mesh M and said simplified mesh M'.

2. Method for the simplification of a source mesh according to claim 1, characterized in that said step of pseudo-optimization comprises a step of enumerating the sharp edges around the two vertices forming the edge to be merged and a step of positioning said resulting vertex, in which the following two cases are distinguished:

- if the numbers of sharp edges are the same around the two vertices, the vertex resulting from the merger is placed in the middle of the segment linking said vertices (42, 44) ;
- if the numbers of sharp edges are different, the vertex resulting from the merger is placed on the vertex with the greatest number of sharp edges (43, 45).

3. Method for the simplification of a source mesh according to any of the claims 1 and 2, characterized in that it comprises a step for the selection of an edge merger to be made among all the edge mergers possible, taking account of :

- at least one piece of information representing the curvature defined locally around the edge considered ;
- at least one piece of information representing the geometrical dynamics defined locally.

4. Method for the simplification of a source mesh according to claim 3, characterized in that said step of selection implements a queue of priorities of edges to be merged as a function of a priority criterion, said information

representing the curvature and then a secondary criterion, said information representing the geometrical dynamics.

5. Method for the simplification of a source mesh according to claim 4, characterized in that said selection step manages a threshold of curvature, only the edges with a curvature below said threshold being considered for the application of said secondary criterion,

said threshold being raised when there is no longer any edge having a curvature below this threshold.

6. Method for the simplification of a source mesh according to any of the claims 1 to 5, characterized in that said information representing the geometrical dynamics belongs to the group comprising :

- the length of the edge considered ;
- a mean of the surfaces of the faces neighboring said edge considered ;
- a mean of the lengths of the edges adjacent to the vertices forming said edge considered ;
- a combination of the lengths of edges and/or surfaces of faces;

7. Method for the simplification of a source mesh according to any of the claims 1 to 6, characterized in that the decimation is interrupted as a function of one of the criteria belonging to the group comprising:

- a compression rate achieved ;
- a geometrical complexity achieved, expressed by a number of vertices or faces ;
- a threshold of curvature achieved.

8. Method for the simplification of a source mesh according to any of the claims 1 to 7, characterized in that it constitutes a step of initialization of a method of geometrical optimization of a mesh.

9. Method for the geometrical optimization of a source mesh comprising a step of initialization implementing the method of simplification according to any of the claims 1 to 7.

10. Method for the encoding of a source mesh (M) according to claim 9, representing a 3D object, delivering a simplified mesh (M') corresponding to said source mesh (M), said meshes being defined by a set of vertices, edges and/or faces,

characterized in that it implements a step of simplification according to any of the claims 1 to 7, and then a step of minimization of a volume contained between said source mesh (M) and said simplified mesh (M').

11. Method for the encoding of a source mesh according to claim 10 characterized in that, each of said meshes being defined by the position of each of its vertices, said minimizing step provides for the determining of the position of the vertices (X_1, X_2, ..., X_n) of said simplified mesh (M') minimizing the volume V(M, M') between said source mesh and said simplified mesh.

12. Method for the encoding of a source mesh according to any of the claims 10 and 11, characterized in that said minimizing step implements an iterative process progressively optimizing the positions of the vertices of said simplified mesh (M').

13. Method for the encoding of a source mesh according to claim 12, characterized in that said iterative process is interrupted when at least one of the following stopping criteria is achieved :

- a maximum number of iterations ;
- a difference between two successive shift vectors of the positions of the vertices that is below a predetermined threshold (ε).

14. Method for the encoding of a source mesh according to any of the claims 11 to 13, characterized in that said step of minimization implements an adaptive gradient method.

15. Method for the encoding of a source mesh according to claim 14, characterized in that said adaptive gradient method relies on the following operations:

- the selection of a vector X_p of $\mathbf{R}^{3n}$ ($n > 1$) of said simplified mesh and the computation of the gradient ∇E (X_p) in X_p of the function to be minimized $E = d(M,M' (X_1,\dots X_n))$:
- the determining of the position of $X_p{}^*$ of X_p of said mesh according to the relationship defined in the iteration k+1 by :

$$X_p{}^{k+1} = X_p{}^k - \gamma_k H \frac{L\,E(X_p{}^k)}{\left\| L\,E(X_p{}^k) \right\|}$$

k varying from 0 to n-1 (with n < N) and γ_k being the step of said relationship.

16. Method for the encoding of a source mesh according to any of the claims 10 to 15 characterized in that, at each iteration, an elementary variation of said volume corresponding to a vector field $\delta\vec{M}$ is determined and in that, since the surface is parametrized by u and v so that a vector $\delta\vec{M}$ is expressed in the form $\delta\vec{M}$ (u, v), said elementary variation is likened to the parallelepiped generated by the evolution of the surface area element dudv in the direction $\delta\vec{M}$ (u, v).

17. Method for the encoding of a source mesh according to any of the claims 10 to 16, characterized in that said simplified mesh is parametrized by means of a model of finite elements.

18. Method for the encoding of a source mesh according to claim 17, characterized in that said finite elements are advantageously obtained by means of a refined interpolator.

19. Method for the encoding of a source mesh according to any of the claims 10 to 18, characterized in that said method implements a progressive encoding of said simplified mesh by decimation and local optimization.

20. Method for the encoding of a source mesh according to any of the claims 10 to 19, characterized in that it comprises a step of limitation of the deterioration due to an elementary conversion implementing a priority queue on the elementary conversions.

21. Method for the encoding of a source mesh according to claim 20, characterized in that said step of limitation of the deterioration due to an elementary conversion, namely an edge merger, defined by two vertices, comprises the steps of :

- computing a cost for each possible elementary conversion ;
- carrying out the lowest cost elementary conversion ;
- recomputing the costs of the elementary conversions modified by the previous elementary conversion ;
- adding the new elementary conversions created and computing the corresponding costs.

22. Method for the encoding of a source mesh according to claim 21, wherein the cost of an elementary conversion (T_i) is expressed by:

$$C(T_i(X_i, X_j)) = \max d_2(V_M, F(X_i^f))$$

with :

T_i conversion merging two vertices X_i and X_j of said simplified mesh M' ;

X_i^f the vertex of said simplified mesh M' resulting from said conversion ;

$F(X_i^f)$ the faces of said simplified mesh M' neighboring the vertex X_i^f after said conversion ;

V_M set of the vertices of said source mesh M belonging to the faces having been intersected during the computation of the orientation of the surfaces during said minimization.

23. Application of the method for the encoding of a source mesh according to any of the claims 1 to 22 to at least to the following fields :

- virtual reality ;
- scientific simulation ;
- modelling.

ABSTRACT OF THE DISCLOSURE

METHOD FOR THE ENCODING OF A SOURCE MESH WITH OPTIMIZATION OF THE POSITION OF A VERTEX RESULTING FROM AN EDGE MERGER, AND CORRESPONDING APPLICATIONS

The invention relates to a method for the simplification of a source mesh (M) implementing a step of decimation by edge merger and comprising a pseudo-optimizing step after said step of decimation by merger of a edge, positioning the vertex resulting from said merger so as to reduce the geometrical deviation between said source mesh M and said simplified mesh M'. Advantageously, said step of pseudo-optimization consists in enumerating the sharp edges around two vertices forming the edge to be merged and distinguishing the following two cases :

- if the numbers of sharp edges are the same around two vertices, the vertex resulting from the merger is placed in the middle of the segment linking said vertices (42, 44) ;
- if the numbers of sharp edges are different, the vertex resulting from the merger is placed on the vertex with the greatest number of sharp edges.

Figure 4.

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Légendes des dessins:

Figure 2:

curvature (rad)

Figure 4:
Initial position
Edge to be merged

Figure 14:
Hauteur reportée : transferred height
Hauteur = height
Origine = Origin
Coté longueur maxi : Max. length side
Echantillonage : Sampling

Figure 15:
Relative orientation of the surfaces

Figure 16:
Vertices to be merged

Figure 17:
191 : Copy : M --→ M'
192 : Search for elementary conversions that can be performed
193 : Computation of the energy costs of the conversions after optimization.
194 : Performance of the lowest cost conversion
195 : Updating of the energy costs of the conversions in the neighborhood affected by the preceding conversion.
196 : Addition of new conversions and computation of the corresponding costs.

Figure 18:

201: Initialization in position
202: Computation of the gradient
203: Shift
204: Changes in the step
205: Convergence test

Figure 23:

Iteration

Figure 25:

Faces to be eliminated
Initial positions

Figure 26:

Conversion to be updated
Modified edge
Merged edge
Vertex to be optimized

Figure 27:

New conversions

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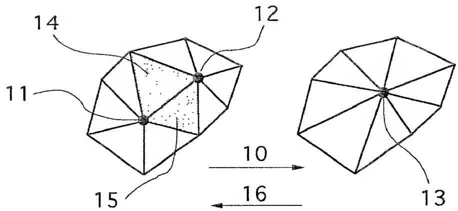


Fig. 1

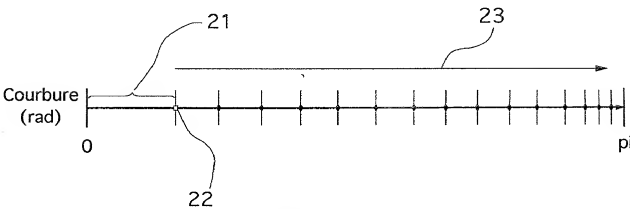


Fig. 2

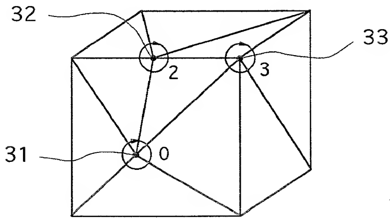


Fig. 3

09/806183

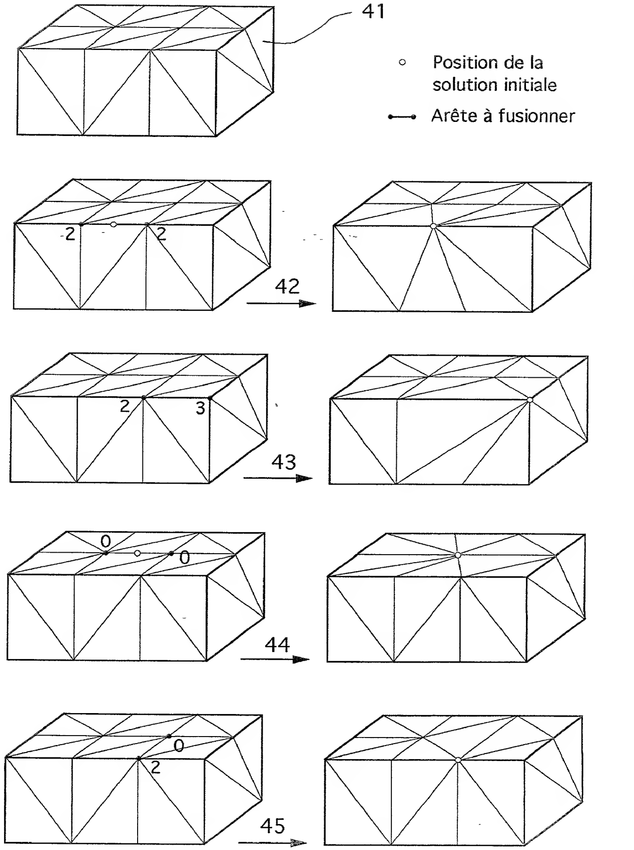


Fig. 4

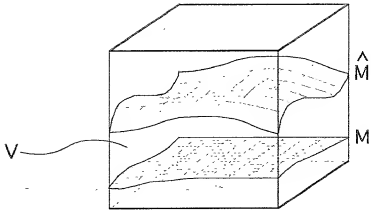


Fig. 5

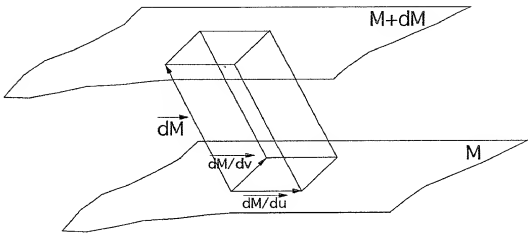


Fig. 6

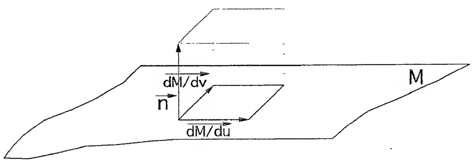


Fig. 7

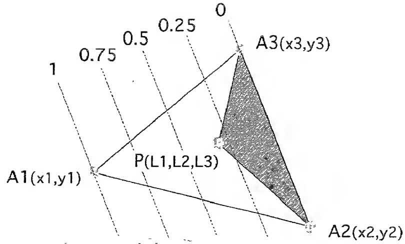


Fig. 8

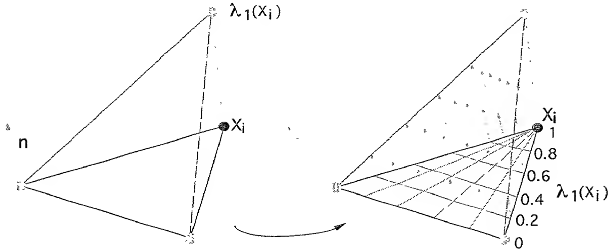


Fig. 9

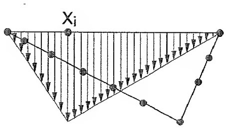


Fig. 10

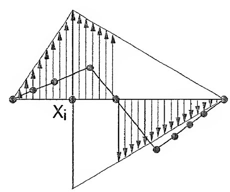
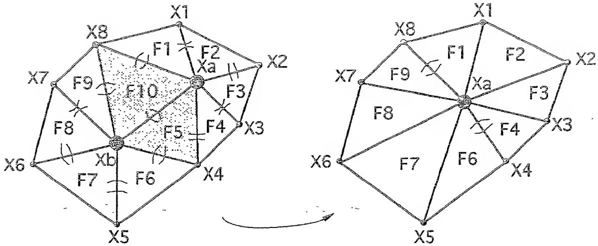


Fig. 11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X8
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F2
X2
Xa
X7
F9
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F3
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F5
F4
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X4
F7
F6
X5
X1
X8
F1
F2
X2
X7
F9
Xa
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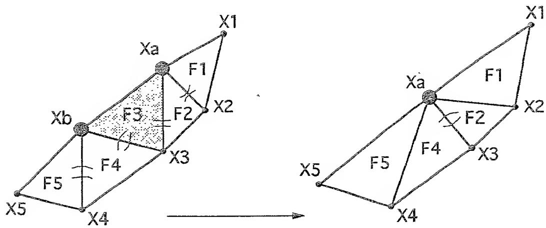
X3
F4
F6
X6
F7
X4
X5

Fig. 12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Xa
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F3
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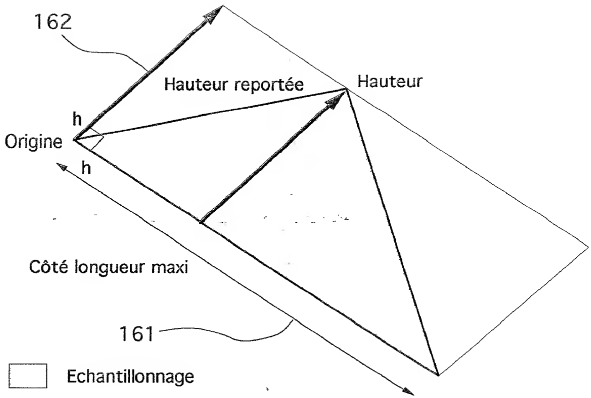
X5
X4

Fig. 13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Hauteur reportée
Hauteur
h
Origine
h
Côté longueur maxi
161
Echantillonnage

Fig. 14

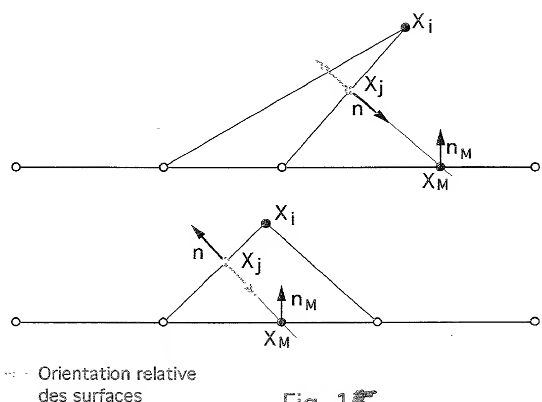

X i
X j
n
n M
X M
X i
n
X j
n M
X M
Orientation relative
des surfaces

Fig. 15

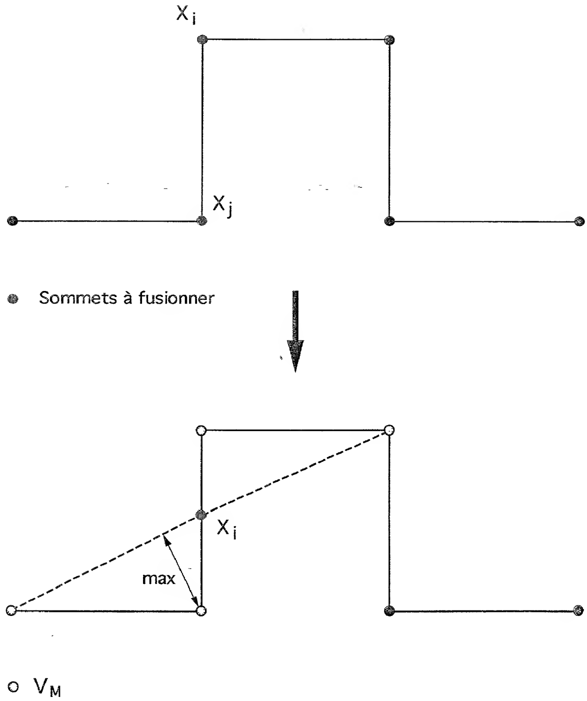


Fig. 16

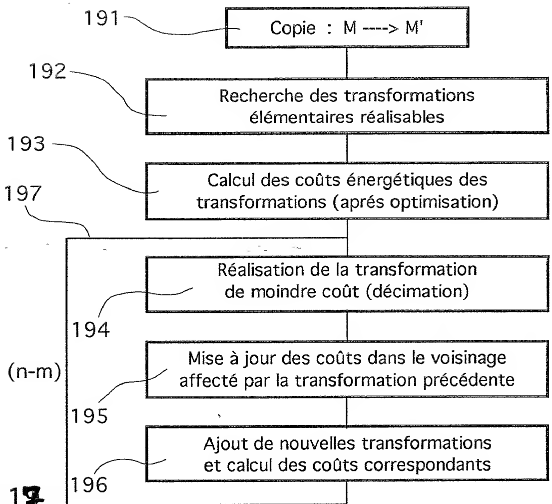
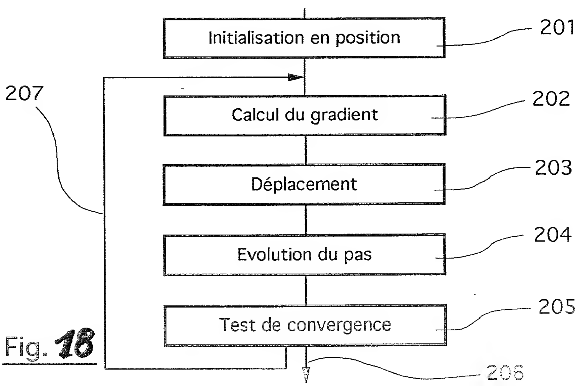

191
Copie : M ----> M'
192
Recherche des transformations
élémentaires réalisables
193
Calcul des coûts énergétiques des
transformations (aprés optimisation)
197
Réalisation de la transformation
de moindre coût (décimation)
194
(n-m)
Mise à jour des coûts dans le voisinage
affecté par la transformation précédente
195
Ajout de nouvelles transformations
et calcul des coûts correspondants
196

Fig. 17


Initialisation en position
201
207
Calcul du gradient
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Déplacement
203
Evolution du pas
204
Test de convergence
205
206

Fig. 18

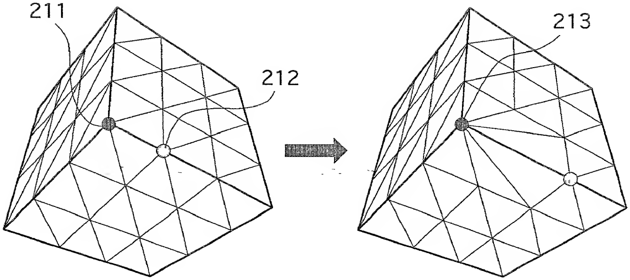

Fig. 19

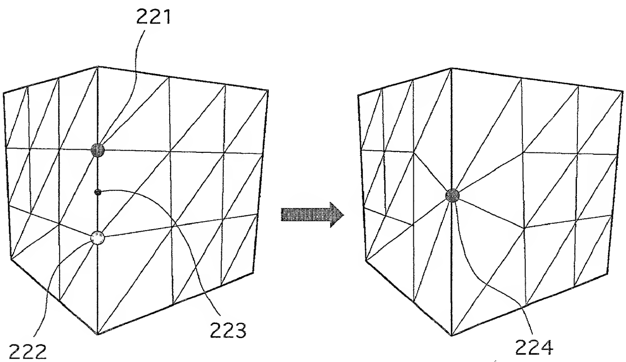

Fig. 20

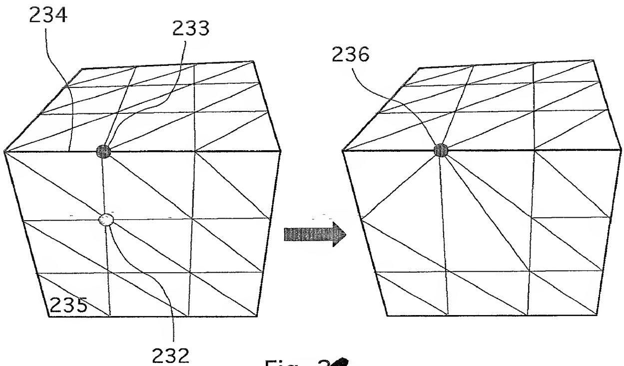

Fig. 21

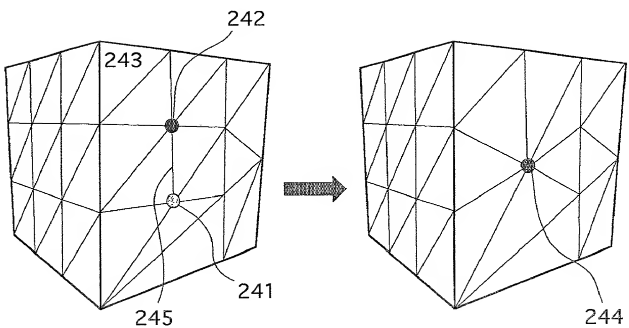

Fig. 22

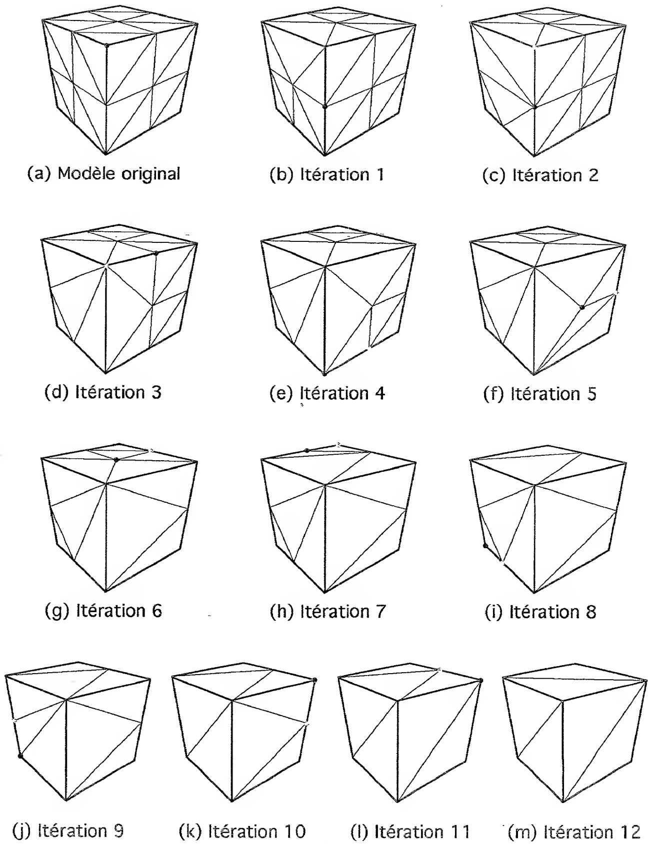
(a) Modèle original
(b) Itération 1
(c) Itération 2
(d) Itération 3
(e) Itération 4
(f) Itération 5
(g) Itération 6
(h) Itération 7
(i) Itération 8
(j) Itération 9
(k) Itération 10
(l) Itération 11
(m) Itération 12

Fig. 23

09/806183

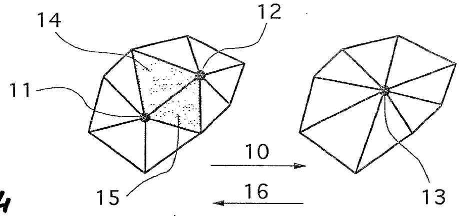

Fig. 24

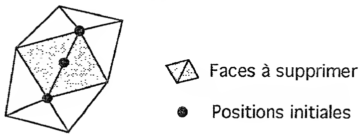

Fig. 25

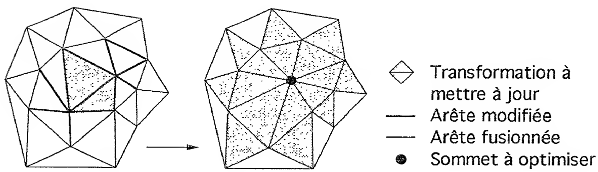

Fig. 26

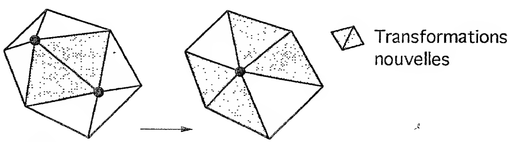

Fig. 27

Attorney Docket No. 9320.125USWO

MERCHANT & GOULD P.C.

United States Patent Application

COMBINED DECLARATION AND POWER OF ATTORNEY

As a below named inventor I hereby declare that: my residence, post office address and citizenship are as stated below next to my name; that

I verily believe I am the original, first and sole inventor (if only one name is listed below) or a joint inventor (if plural inventors are named below) of the subject matter which is claimed and for which a patent is sought on the invention entitled: METHOD FOR THE ENCODING OF A SOURCE MESH WITH OPTIMIZATION OF THE POSITION OF A VERTEX RESULTING FROM AN EDGE MERGER, AND CORRESPONDING APPLICATIONS

The specification of which
a. ☐ is attached hereto
b. ☒ was filed on as application serial no. and was amended on (if applicable) (in the case of a PCT-filed application) described and claimed in international no. PCT/FR99/02524 filed October 15, 1999 and as amended on October 20, 2000 (if any), which I have reviewed and for which I solicit a United States patent.

I hereby state that I have reviewed and understand the contents of the above-identified specification, including the claims, as amended by any amendment referred to above.

I hereby claim foreign priority benefits under Title 35, United States Code, § 119/365 of any foreign application(s) for patent or inventor's certificate listed below and have also identified below any foreign application for patent or inventor's certificate having a filing date before that of the application on the basis of which priority is claimed:

a. ☐ no such applications have been filed.
b. ☒ such applications have been filed as follows:

FOREIGN APPLICATION(S), IF ANY, CLAIMING PRIORITY UNDER 35 USC § 119			
COUNTRY	**APPLICATION NUMBER**	**DATE OF FILING (day, month, year)**	**DATE OF ISSUE (day, month, year)**
France	98 13090	15 October 1998	
France	99 00304	11 January 1999	

ALL FOREIGN APPLICATION(S), IF ANY, FILED BEFORE THE PRIORITY APPLICATION(S)			
COUNTRY	**APPLICATION NUMBER**	**DATE OF FILING (day, month, year)**	**DATE OF ISSUE (day, month, year)**

I hereby claim the benefit under Title 35, United States Code, § 120/365 of any United States and PCT international application(s) listed below and, insofar as the subject matter of each of the claims of this application is not disclosed in the prior United States application in the manner provided by the first paragraph of Title 35, United States Code, § 112, I acknowledge the duty to disclose material information as defined in Title 37, Code of Federal Regulations, § 1.56(a) which occurred between the filing date of the prior application and the national or PCT international filing date of this application.

U.S. APPLICATION NUMBER	**DATE OF FILING (day, month, year)**	**STATUS (patented, pending, abandoned)**

I hereby claim the benefit under Title 35, United States Code § 119(e) of any United States provisional application(s) listed below:

U.S. PROVISIONAL APPLICATION NUMBER	**DATE OF FILING** (Day, Month, Year)

I acknowledge the duty to disclose information that is material to the patentability of this application in accordance with Title 37, Code of Federal Regulations, § 1.56 (reprinted below):

§ 1.56 Duty to disclose information material to patentability.

(a) A patent by its very nature is affected with a public interest. The public interest is best served, and the most effective patent examination occurs when, at the time an application is being examined, the Office is aware of and evaluates the teachings of all information material to patentability. Each individual associated with the filing and prosecution of a patent application has a duty of candor and good faith in dealing with the Office, which includes a duty to disclose to the Office all information known to that individual to be material to patentability as defined in this section. The duty to disclose information exists with respect to each pending claim until the claim is canceled or withdrawn from consideration, or the application becomes abandoned. Information material to the patentability of a claim that is canceled or withdrawn from consideration need not be submitted if the information is not material to the patentability of any claim remaining under consideration in the application. There is no duty to submit information which is not material to the patentability of any existing claim. The duty to disclose all information known to be material to patentability is deemed to be satisfied if all information known to be material to patentability of any claim issued in a patent was cited by the Office or submitted to the Office in the manner prescribed by §§ 1.97(b)-(d) and 1.98. However, no patent will be granted on an application in connection with which fraud on the Office was practiced or attempted or the duty of disclosure was violated through bad faith or intentional misconduct. The Office encourages applicants to carefully examine:

(1) prior art cited in search reports of a foreign patent office in a counterpart application, and

(2) the closest information over which individuals associated with the filing or prosecution of a patent application believe any pending claim patentably defines, to make sure that any material information contained therein is disclosed to the Office.

(b) Under this section, information is material to patentability when it is not cumulative to information already of record or being made of record in the application, and

(1) It establishes, by itself or in combination with other information, a prima facie case of unpatentability of a claim;

or

(2) It refutes, or is inconsistent with, a position the applicant takes in:

(i) Opposing an argument of unpatentability relied on by the Office, or

(ii) Asserting an argument of patentability.

A prima facie case of unpatentability is established when the information compels a conclusion that a claim is unpatentable under the preponderance of evidence, burden-of-proof standard, giving each term in the claim its broadest reasonable construction consistent with the specification, and before any consideration is given to evidence which may be submitted in an attempt to establish a contrary conclusion of patentability.

(c) Individuals associated with the filing or prosecution of a patent application within the meaning of this section are:

(1) Each inventor named in the application:

(2) Each attorney or agent who prepares or prosecutes the application; and

(3) Every other person who is substantively involved in the preparation or prosecution of the application and who is associated with the inventor, with the assignee or with anyone to whom there is an obligation to assign the application.

(d) Individuals other than the attorney, agent or inventor may comply with this section by disclosing information to the attorney, agent, or inventor.

(e) In any continuation-in-part application, the duty under this section includes the duty to disclose to the Office all information known to the person to be material to patentability, as defined in paragraph (b) of this section, which became available between the filing date of the prior application and the national or PCT international filing date of the continuation-in-part application.

I hereby appoint the following attorney(s) and/or patent agent(s) to prosecute this application and to transact all business in the Patent and Trademark Office connected herewith:

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I hereby authorize them to act and rely on instructions from and communicate directly with the person/assignee/attorney/firm/ organization who/which first sends/sent this case to them and by whom/which I hereby declare that I have consented after full disclosure to be represented unless/until I instruct Merchant & Gould P.C. to the contrary.

I understand that the execution of this document, and the grant of a power of attorney, does not in itself establish an attorney-client relationship between the undersigned and the law firm Merchant & Gould P.C., or any of its attorneys.

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P.O. Box 2903
Minneapolis, MN 55402-0903

23552
PATENT TRADEMARK OFFICE

I hereby declare that all statements made herein of my own knowledge are true and that all statements made on information and belief are believed to be true; and further that these statements were made with the knowledge that willful false statements and the like so made are punishable by fine or imprisonment, or both, under Section 1001 of Title 18 of the United States Code and that such willful false statements may jeopardize the validity of the application or any patent issued thereon.

2	**Full Name Of Inventor**	**Family Name** Laurent-Chatenet	**First Given Name** Nathalie	**Second Given Name**
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0	**Residence & Citizenship**	**City** Cesson Sevigne	**State or Foreign Country** France	**Country of Citizenship** France
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Signature of Inventor 203:			**Date:** 02/05/01	